DOCUMENT RESUME

ED 327 402 SE 051 849

TITLE	Mathematics Curriculum Guide General Mathematics. Bulletin 1987, No. 50.
INSTITUTION	Alabama State Dept. of Education, Montgomery.
PUB DATE	87
NOTE	402p.; For related documents, see SE 051 846-852.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052) --
EDRS PRICE	MF01/PC17 Plus Postage.
DESCRIPTORS	Computation; Consumer Education; Course Content; Equations (Mathematics); Estimation (Mathematics); Fractions; Geometry; Instructional Materials; *Learning Activities; Mathematical Concepts; *Mathematics Education; Mathematics Skills; Metric System; Numbers; Probability; *Problem Solving; Secondary Education; *Secondary School Mathematics; Standardized Tests; State Curriculum Guides; Statistics; *Teaching Methods
IDENTIFIERS	Alabama

ABSTRACT

This guide is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12. It is inclusive also of the objectives tested by the Stanford Achievement Tests and the Alabama Basic Competency Tests. One characteristic of the curriculum guide is that it clearly states what students should learn in each grade level. These objectives are called student outcomes and appear in the table of contents. The majority of the material, however, consists of classroom activities that lead students to the desired outcomes. The curriculum guide specifies what to teach as well as ways to teach it. Each section of the guide contains a specific table of contents, which lists what students should learn; activities; original sources; and student outcomes. Activities are preceded by a description of the area of mathematics covered, the specific student outcomes that are addressed in the activities, and notes for the teacher, which convey information important for effectively carrying out the activities. Topics include computation, estimation, approximation, powers, square roots, decimals, fractions, percent, integers, geometry, linear equations, inequalities, metric system, measurement, probability, statistics, and consumer mathematics. (KR)

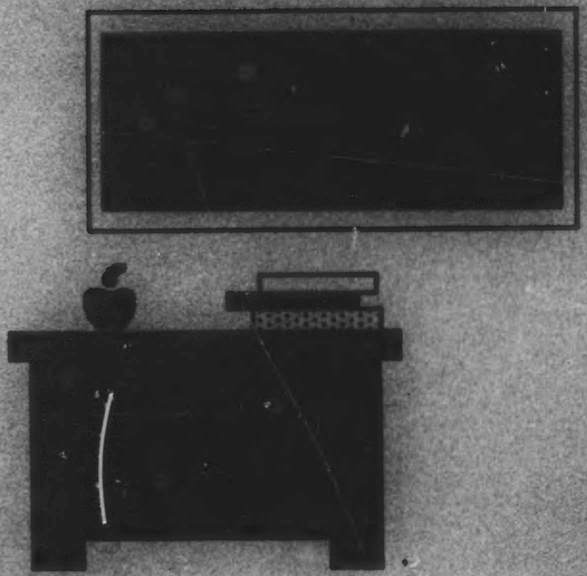
****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

SE051849

ED327402

MATHEMATICS

CURRICULUM
GUIDE
GENERAL MATHEMATICS



Bulletin 1987, No. 50

WAYNE TEAGUE
State Superintendent of Education
Alabama State Department of Education
Montgomery, Alabama

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
W. Teague

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE

It is the official policy of the Alabama State Department of Education, including postsecondary institutions under the control of the State Board of Education, that no person in Alabama shall, on the grounds of race, color, handicap, sex, religion, creed, national origin, or age, be excluded from participation in, be denied the benefits of, or be subject to discrimination under any program, activity, or employment.

MATHEMATICS
CURRICULUM GUIDE
GENERAL MATHEMATICS

ALABAMA STATE DEPARTMENT OF EDUCATION
Wayne Teague
State Superintendent of Education

DIVISION OF STUDENT INSTRUCTIONAL SERVICES
Eldon D. Johnson
Director
Kennith S. Blankenship
Assistant Director
Richard Causey
Assistant Director

CURRICULUM DEVELOPMENT SECTION
Linda G. Harris
Project Director

FOREWORD

This Mathematics Curriculum Guide: General Mathematics is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12 (Bulletin 1984, No. 16). It is inclusive also of the objectives tested by the Stanford Achievement Test and the Alabama Basic Competency Test.

One characteristic of the curriculum guide is that it clearly states what students should learn in General Math. These are called STUDENT OUTCOMES and appear in the Table of Contents. The majority of the material, however, consists of classroom ACTIVITIES that lead students to the desired outcomes. The curriculum guide, then, specifies what to teach as well as ways to teach it.

The Mathematics Curriculum Guide: General Mathematics is supplemental in nature. It will add to material found in textbooks and complement locally developed skills continuums, scope and sequences, and/or curriculum guides. In the past, we have been most successful when we clearly defined what students should learn, gave examples of how it can be taught, and then tested for mastery. This curriculum guide is another step in that direction.

Wayne Teague

Wayne Teague
State Superintendent of Education

INTRODUCTION

The Mathematics Curriculum Guide: General Mathematics is one of a series of curriculum guides to be developed by the Alabama State Department of Education. Its production is part of a larger plan to provide LEAs with a curriculum guide in every subject area K-12, immediately following the Course of Study adoption in the same subject area. The Mathematics Curriculum Guide K-6 is packaged separately. Each of the other secondary mathematics guides (Math - Grades 7 and 8, General Math, Consumer Math, Pre-Algebra, Geometry, Algebra II, Trigonometry/Analysis, and Applied Vocational Math will be packaged separately also and will be published shortly.

The General Mathematics guide has three distinct sections. It begins with a TABLE OF CONTENTS which lists what students should learn (student outcomes) in General Math. After each outcome, a page number(s) directs the reader to classroom activities that address the specific outcome.

The second and largest section of the guide consists of classroom ACTIVITIES. Activities are preceded by a description of the area of General Math covered, the specific student outcomes that are addressed in the activities, and Notes for the Teacher that convey information important for effectively carrying out the activities.

The third section is referred to as the ORIGINAL SOURCES AND STUDENT OUTCOMES portion of the guide. It summarizes the relationship of student outcomes to the original sources from which the outcomes were taken. (The original sources for the Mathematics Curriculum Guide: General Mathematics outcomes were the Alabama Course of Study, the Stanford Achievement Test objectives, and the Alabama Basic Competency Skills.) This section of the guide will serve as a reference for local school systems that may wish to trace the source of each outcome or to rate each outcome as being basic, standard, or advanced.

The use of the Mathematics Curriculum Guide: General Mathematics is not mandatory. Instead, it is offered as material that will help implement the Mathematics Course of Study; ensure curriculum validity for the Basic Competency Program; and prepare students for the Stanford Achievement Tests. The Mathematics Curriculum Guide: General Mathematics can be used as it is in LEAs that currently have no guide, or it can serve as a starting point for developing a similar guide locally. In either case, the involvement of local educators in the development of curricula is valued and encouraged. It is hoped that this guide will bring about local expansion activities such as writing outcomes for "optional" mathematics courses; adding to the list of outcomes for a given subject; writing additional activities in specific areas of identified weakness; rating outcomes as basic, standard, or advanced; and developing uniform end-of-unit tests.

ACKNOWLEDGEMENTS

From its inception, the development of curriculum guides has been viewed as a cooperative effort between specialists in the Division of Student Instructional Services and selected classroom teachers from the public schools of Alabama. The Curriculum Development Section, under the leadership of Katherine Mitchell, has the responsibility for coordinating this project.

Work on the Mathematics Curriculum Guide: General Mathematics started in January 1986, when Linda G. Harris was selected as Project Director and the identification of student outcomes began. The second phase of the project took place during July and August when five classroom teachers joined State Department of Education specialists in the writing of activities that teach the identified outcomes. For twelve days the following teachers offered their best ideas:

James Davidson - Jasper City Schools
Patricia Edwards - Etowah County Schools
Myrtle Johnson - Eufaula City Schools
Ann Lilly - Tuscaloosa City Schools
Wanda Motes - Enterprise City Schools

Over a period of eleven months, a team of State Department of Education specialists identified and reviewed student outcomes, wrote and rewrote activities, read activities for clarity, and proofed various drafts. Robin Lanier contributed the graphics for the guide. Others who were directly involved with the Mathematics Curriculum Guide: General Mathematics were Kay Atchison, Jack Blucert, Moses Clark, Wanda Coleman, Frank Heatherly, and Mary Jane Herring. The General Math guide was the last document that our friend and co-worker, Jim Smith, contributed to before his death on December 4, 1986. Brondena Ferguson of the Student Assessment Section typed all 202 pages of this guide. She gave it the same care, attention, and love that she gives her primary responsibilities. We appreciate the work of all these people and their co-workers, who made the adjustments and sacrifices necessary to carry out their own programs and, at the same time, made the Mathematics Curriculum Guide: General Mathematics a division product.

GENERAL MATHEMATICS

Table of Contents

PAGE

WHOLE NUMBERS: COMPUTATION

1. Solve addition problems with no more than five addends. ---------- 15-21
2. Solve addition problems with addends up to five digits. ---------- 15-21
3. Solve addition problems with or without regrouping. ---------- 15-21
4. Compute sums presented in horizontal or vertical format. ---------- 15-21
5. Subtract numbers with subtrahends and minuends of up to five digits with or without regrouping. ------ 15-21
6. Solve subtraction problems presented in horizontal or vertical format. ---------- 15-19
7. Multiply one-, two-, and three-digit factors. ---------- 15-21
8. Solve multiplication problems presented in horizontal or vertical format. ---------- 15-19
9. Solve division problems with dividends up to five digits with or without remainders. ---------- 15-21
10. Solve division problems presented symbolically by: a ÷ b or b/a. ---------- 15-19
11. Interpret and solve division problems stated in fractional format, i.e., $\frac{a}{b}$. ---------- 15-19

WHOLE NUMBERS: ESTIMATION AND APPROXIMATION

12. Interpret place value in numerals of up to seven digits and round numbers. ---------- 22-25
13. Estimate sums, differences, products, and quotients of whole numbers. ---------- 22-25
14. Estimate a sum with up to five addends. ---------- 22-25
15. Round whole numbers to tens, hundreds, and thousands. ---------- 22-25
16. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients. ---------- 22-25

WHOLE NUMBERS: POWERS AND SQUARE ROOTS

17. Read powers of numbers and find the products they indicate. ---------- 20-21, 26-28
18. Identify the expanded form of a number expressed in exponential form. ---------- 26-28
19. Subtract a number less than ten from a number expressed in exponential form. ---------- 26-28

GENERAL MATHEMATICS

Table of Contents

PAGE

WHOLE NUMBERS: POWERS AND SQUARE ROOTS (Cont´d)

20. Identify the square roots of perfect squares. ---------- 20-21, 26-28
21. Find square roots using a table of square roots. ---------- 26-28

WHOLE NUMBERS: APPLICATIONS

22. Solve one-step and two-step problems. ---------- 90-91
23. Solve word problems. ---------- 90-91
24. Solve rate problems. ---------- 160
25. Identify the number sentence appropriate to solve a two-step problem. ---------- 90-91
26. Identify the information needed to solve a problem. ---------- 90-91
27. Identify the information not needed to solve a problem. ---------- 90-91
28. Solve a problem requiring the calculation of an average. ---------- 160
29. Solve word problems to determine miles per gallon when given distance in miles and gallons of fuel used. ---------- 160
30. Solve word problems to determine time when given distance traveled and average rate of speed. ---------- 160
31. Solve word problems to determine rate when given distance traveled and time. ---------- 160
32. Solve word problems involving computation of net pay when salary and deductions are given in only dollar amounts. ---------- 160
33. Solve word problems involving computation of net pay when amounts of deductions, the rate of pay, and the number of hours worked are given. ---------- 160
34. Solve word problems requiring the interpretation of a picto-, bar, line, or circle graphs and the use of information contained in the graphs. ---------- 160

DECIMALS: COMPARE AND ORDER

1. Identify place value in decimal numbers and mixed decimals up to six digits. ---------- 29-31
2. Compare decimal numbers between zero and one to thousandths. ---------- 29-31
3. Select from the options a decimal less than or greater than a given decimal. ---------- 29-31

GENERAL MATHEMATICS

Table of Contents

PAGE

DECIMALS: COMPUTATION

4. Add two or more decimal numbers. 15-19
5. Add decimal numbers and whole numbers. 15-19
6. Add decimal numbers and mixed decimals. 15-19
7. Add whole numbers and mixed decimals. 15-19
8 Add two or more mixed decimals. 15-19
9. Add sums involving up to three unlike decimal addends. 15-19
10. Add decimal numbers to two places with regrouping and with no more than five digits to the left of the decimal. 15-19
11. Add sums involving money values. 15-19
12. Compute sums presented in horizontal or vertical format. 15-19
13. Subtract one decimal number from another. 15-19
14. Subtract a whole number from a decimal number and vice versa. 15-19
15. Subtract a decimal number from a mixed decimal. 15-19
16. Subtract one mixed decimal from another. 15-19
17. Subtract decimal numbers to two places with regrouping. 15-19
18. Subtract money values. 15-19
19. Subtract decimal numbers with no more than five digits to the left of the decimal. 15-19
20. Solve subtraction problems presented in horizontal or vertical format. 15-19
21. Multiply: whole numbers and decimal numbers; whole numbers and mixed decimals; a decimal number by another decimal number; decimal numbers and mixed decimals; and a mixed decimal by another mixed decimal. 15-19
22. Multiply a decimal by a power of ten by relocating the decimal point. 15-19
23. Multiply hundredths times tenths. 15-19
24. Multiply hundredths times hundredths. 15-19
25. Multiply money values. 15-19
26. Multiply decimals with no more than three digits to the left of the decimal. 15-19
27. Multiply decimal numbers presented in horizontal or vertical format. 15-19
28. Divide a decimal number or a mixed decimal by a whole number, correctly placing the decimal point in the quotient. 15-19
29. Express the quotient of two whole numbers as a decimal number. 15-19

GENERAL MATHEMATICS

Table of Contents

PAGE

DECIMALS: COMPUTATION (Cont'd)

30. Divide by powers of ten by relocating the decimal point. ---------- 15-19
31. Rewrite and solve a division problem with a decimal number or a mixed decimal divisor as an equivalent problem with a whole number. ---------- 15-19
32. Divide hundredths by tenths. ---------- 15-19
33. Divide tenths by whole numbers of not more than two digits. ---------- 15-19
34. Divide decimal numbers with no more than two digits to the left of the decimal in the divisor. ------- 15-19
35. Divide decimal numbers with dividends not to exceed five digits. ---------- 15-19

DECIMALS: CONVERSION TO FRACTIONS AND PERCENTS

36. Select a whole percent up to 200% equivalent to a given decimal number up to two places. ---------- 15-19
37 Select a decimal number up to two places, equivalent to a given percent, up to 200%. ---------- 15-19
38. Rename a decimal fraction or a mixed decimal, up to three digits, as a percent or vice versa. -------- 157-158
39. Select a fraction or a mixed number equivalent to a given decimal. ---------- 157-158
40. Select a decimal equivalent to a given fraction or mixed number. ---------- 157-158

DECIMALS: ESTIMATION AND APPROXIMATION

41. Identify place value in decimal numbers and mixed decimals up to six digits. ---------- 22-25
42. Round decimal numbers and mixed decimals to a specified place value. ---------- 22-25
43. Estimate to the nearest whole number sums, differences, products, and quotients involving decimal numbers or mixed decimals. ---------- 22-25
44. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients. ---------- 22-25
45. Estimate to the nearest ten cents or dollar. ---------- 22-25
46. Round to the nearest dollar. ---------- 22-25

GENERAL MATHEMATICS

Table of Contents

PAGE

DECIMALS: APPLICATIONS

47. Solve word problems involving realistic applications requiring addition, subtraction, multplication, or division of decimals including metric or money values. 161
48. Solve word problems requiring the calculation of final cost when the discount or sales tax rate and the items' costs are given. 161
49. Find amount of discount, commission, or down payment when the rate and total cost are given. 161
50. Find averages to hundredths. 161
51. Solve word problems involving a total and the number of items totaled, and requiring the calculation of the average to hundredths. 161
52. Solve word problems involving time. 161
53. Solve word problems using information obtained from tables and charts. 161
54. Determine change from a given amount of money on a purchase of two or more items. 161
55. Solve word problems describing a realistic situation with no more than five items, addends up to $99.99, and amount tendered up to $200.00. 161
56. Solve problems comparing unit costs and rates to determine the best purchase or determine the amount of saving when the same item is offered at different prices. 161
57. Solve word problems that compare the cost of cash and credit purchases. 161

FRACTIONAL NUMBERS: COMPARE AND ORDER

1. Identify the fraction that is equivalent to a given common fraction. 32-39
2. Identify the common fraction that is expressed in simplest form. 32-39
3. Rename a common fraction to simplest form. 32-39
4. Rename a mixed number as an improper fraction. 32-39
5. Find the least common denominator for three common fractions. 40-45
6. Identify the greatest of four common fractions with unlike denominators. 40-45
7. Order three common fractions with unlike denominators from least to greatest. 40-45
8. Identify the proper fractions that are less than or greater than a given proper fraction. 40-45

GENERAL MATHEMATICS

Table of Contents

PAGE

FRACTIONAL NUMBERS: COMPUTATION

9. Add two common fractions with like denominators with renaming. ---------- 46-51
10. Add three common fractions with unlike denominators. ---------- 46-51
11. Add two mixed numbers with unlike denominators. ---------- 46-51
12. Add proper fractions and mixed numbers with unlike denominators. ---------- 46-51
13. Subtract one mixed number from another with a like denominator with renaming. ---------- 46-51
14. Subtract a common fraction from a mixed number, each with a different denominator. ---------- 46-51
15. Subtract proper fractions and mixed numbers with unlike denominators. ---------- 46-51
16. Multiply a common fraction by a whole number. ---------- 52-57
17. Multiply two common fractions with renaming. ---------- 52-57
18. Multiply two mixed numbers. ---------- 52-57
19. Multiply two factors of proper fractions and/or mixed numbers. ---------- 52-57
20. Divide a whole number by a common fraction. ---------- 58-61
21. Divide a common fraction by a whole number. ---------- 58-61
22. Divide a mixed number by a whole number. ---------- 58-61

FRACTIONAL NUMBERS: CONVERSION TO DECIMALS AND PERCENTS

23. Rename a common fraction expressed in thousandths as a decimal fraction. ---------- 62-65
24. Rename a mixed decimal as a mixed number in simplest form. ---------- 62-65
25. Rename a decimal fraction as a common fraction in simplest form. ---------- 62-65
26. Rename a percent as a common fraction in simplest form. ---------- 62-65
27. Rename a fraction with denominators 2, 4, 5, 8, 10, 100, or 1000 or a mixed number less than ten to a given decimal. ---------- 62-65
28. Rename a decimal equivalent to a given fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 to a mixed number less than ten. ---------- 62-67

GENERAL MATHEMATICS

Table of Contents

PAGE

FRACTIONAL NUMBERS: ESTIMATION AND APPROXIMATION

29. Make approximations by rounding to whole numbers, proper fractions stated in simplest form and with single digit denominators. 68-71
30. Make approximations by rounding to whole numbers, mixed numbers stated in simplest form and with single digit denominators. 68-71
31. Estimate sums, differences, products, and quotients involving fractional numbers. 68-71
32. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients. 68-71

FRACTIONAL NUMBERS: BASIC OPERATIONS

33. Change a mixed number to an improper fraction. 32-39
34. Change an improper fraction to a mixed number. 32-39
35. Add two or more like or unlike mixed numbers with or without renaming. 46-51
36. Add like or unlike mixed numbers and fractions or whole numbers with or without renaming. 46-51
37. Subtract a mixed number, a fraction, or a whole number from a like or unlike mixed number with or without renaming. 46-51
38. Subtract a mixed number or a proper fraction from a whole number with or without renaming. 46-51
39. Multiply mixed numbers with or without renaming. 52-57
40 Divide mixed numbers with or without renaming. 58-61

FRACTIONAL NUMBERS: APPLICATIONS

41. Recognize the proper operation needed to solve practical problems involving addition, subtraction, and multiplication of fractions. 72-77
42. Solve practical problems involving addition, subtraction, and multiplication of proper fractions and mixed numbers. 72-77
43. Solve a one-step division problem in which the answer is expressed as a fraction in simplest form. 72-77
44. Solve a rate problem involving a fraction or a mixed number. 72-80

GENERAL MATHEMATICS

Table of Contents

PAGE

INTEGERS: MEANING AND REPRESENTATION

1. Demonstrate an understanding of the definition by listing examples of the set of integers. — 81-83
2. Given a number line, identify specific integers. — 81-83
3. Given an integer, state its opposite. — 81-83
4. Define absolute value as the distance a number is from zero on the number line. — 81-83
5. State the integer corresponding to a given absolute. — 81-83
6. Simplify expressions that involve absolute value. — 81-83
7. Identify the integer that is between two given integers. — 81-83
8. Write inequalities using positive and negative integers. — 81-83

INTEGERS: COMPUTATION

9. Add positive integers. — 84-89
10. Add negative integers. — 84-89
11. Add positive and negative integers. — 84-89
12. Subtract positive integers. — 84-89
13. Subtract negative integers. — 84-89
14. Subtract positive and negative integers. — 84-89
15. Multiply positive integers. — 84-89
16. Multiply positive and negative integers. — 84-89
17. Multiply negative integers. — 84-89
18. Divide positive integers. — 84-89
19. Divide a positive integer by a negative integer and vice versa. — 84-89
20. Divide negative integers. — 84-89

INTEGERS: APPLICATIONS

21. Translate word problems into expressions using integers. — 90-92
22. Solve word problems involving integers. — 90-92

GENERAL MATHEMATICS

Table of Contents

PAGE

INFORM GEOMETRY: TWO AND THREE DIMENSIONS

1. Identify and name points, lines, planes, segments, rays, intersecting lines, and parallel lines. ----- 92-93
2. Measure and classify angles as acute, right, or obtuse. ---------- 94-97
3. Identify congruent polygons and corresponding parts of the congruent polygons. ---------- 94-97
4. Classify triangles in terms of their sides and angles. ---------- 94-97
5. Classify polygons in terms of their sides and angles. ---------- 94-97
6. Recongize that the sum of the measures of the angles of a triangle is 180°. ---------- 94-97
7. Identify center, chord, diameter, radius, arc, central angle, and inscribed angle of a circle. ------- 92-93
8. Select the figure or state the names of the following plane geometric figures or lines: angle, right angle, triangle, square, rectangle, circle, lines, perpendicular lines, or parallel lines. ---------- 94-97
9. Identify prisms, pyramids, cylinders, cones, and spheres. ---------- 98-99
10. Select the figure or state the name of the following solid geometric figures: sphere, cube, rectangular solid, cylinder, and cone. ---------- 100-104

INFORMATL GEOMETRY: APPLICATIONS

11. Evaluate formulas. ---------- 100-104
12. Find the perimeter or circumference of a shape. ---------- 100-104
13. Find the perimeter of polygons up to six sides and units expressed in inches, feet, yards, centimeters, or meters. ---------- 100-104
14. Find the perimeter of rectangles, squares, and triangles expressed in word problems. ---------- 100-104
15. Find the perimeter of polygons when all dimensions of the figure are given. ---------- 100-104

LINEAR EQUATIONS AND INEQUALITIES: SOLVING EQUATIONS

1. Use the standard order of operations to evaluate an expression involving more than one operation. ---- 105-108
2. Evaluate an expression for a given value of the variable. ---------- 105-108
3. Evaluate a linear equation. ---------- 105-108

GENERAL MATHEMATICS

Table of Contents

PAGE

LINEAR EQUATIONS AND INEQUALITIES: SOLVING EQUATIONS (Cont'd)

4. Solve an addition or subtraction equation by subtracting the same number from both sides or by adding the same number to both sides. ---- 105-108
5. Solve a multiplication or division equation by dividing or multipling both sides by the same number. - 105-108
6. Solve two-step equations. ---- 105-108
7. Write an expression involving a variable for a given mathematical phrase or situation. ---- 109-110
8. Solve problems by writing equations involving addition, subtraction, multiplication, or division and solving the equation. ---- 109-110
9. Graph inequalities on the number line. ---- 111-113
10. Solve an addition or subtraction inequality involving rational numbers. ---- 111-113
11. Solve a multiplication or a division inequality involving rational numbers. ---- 111-113

MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

1. Recognize equivalent customary linear units. ---- 114-116
2. Recognize equivalent metric linear units. ---- 114-116
3. Measure line segments or objects in a figure in customary and metric units. ---- 117-121
4. Measure line segments or objects in a figure in customary and metric units requiring the addition of two lengths. ---- 117-121
5. Find the area of a rectangle, square, parallelogram, triangle, trapezoid, and a circle. ---- 122-129
6. Find the surface area of a prism, a pyramid, a cylinder, a cone, and a sphere. ---- 122-129
7. Solve word problems relating to rectangular area involving customary or metric units of inches, feet, yards, centimeters, or meters. ---- 122-129
8. Solve problems requiring the calculation of rectangular area when figures are used and lengths of all sides are labeled. ---- 122-129
9. Find the volume of a prism, a pyramid, a cylinder, a cone, and a sphere when the dimensions are given in customary or metric units. ---- 122-129
10. Recognize equivalent customary mass units. ---- 114-116
11. Recognize equivalent metric mass units. ---- 114-116

GENERAL MATHEMATICS

Table of Contents

PAGE

MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS (Cont´d)

12. Choose the appropriate customary measure of mass and find equal measures for given measures of mass. - 114-116
13. Choose the appropriate metric measure of mass and find equal measures for given measures of mass. ---- 114-116
14. Add and subtract with metric measures. (mass) ---- 130-136
15. Add and subtract with customary measures. (mass) ---- 130-136
16. Recognize equivalent customary capacity units. ---- 114-116
17. Recognize equivalent metric capacity units. ---- 114-116
18. Choose the appropriate customary measure of capacity and find equal measures for given measures of capacity. ---- 114-116
19. Choose the appropriate metric measure of capacity and find equal measures for given measures of capacity. ---- 114-116
20. Add and subtract with metric measures. (capacity) ---- 130-136
21. Add and subtract with customary measures. (capacity) ---- 130-136
22. Solve word problems involving capacities with units of ounces, pints, quarts, gallons, milliliters, or liters. ---- 130-136
23. Solve word problems involving capacities but without conversions between metric and customary units without using formulas to determine volume. ---- 130-136
24. Identify the measuring scale for temperature in metric measure as degrees Celsius. ---- 117-121
25. Identify the measuring scale for temperature in customary measure as degrees Fahrenheit. ---- 117-121
26. Add and subtract with metric measures. (temperature) ---- 130-136
27. Add and subtract with customary measures. (temperature) ---- 130-136
28. Solve word problems involving computation of metric measures or customary measures. ---- 130-136
29. Choose the appropriate metric measure for linear, mass, or capacity measure, for a given situation. -- 114-116
30 Choose the appropriate customary mearure for linear, mass, or capacity measure, for a given situation. ---- 114-116
31. Solve problems by giving sensible answers. ---- 130-136
32. Solve word problems using the appropriate customary or metric measures. ---- 130-136
33. Add and subtract measures of time and compute elapsed time. ---- 130-136
34. Solve problems using addition, subtraction, multiplication, and division for units of time to include years, months, weeks, days, hours, and minutes. ---- 130-136
35. Solve problems using fractional units of time involving denominators of 2, 3, 4, 6, 10, and 12. ------ 130-136

GENERAL MATHEMATICS

Table of Contents

PAGE

MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS (Cont´d)

36. Change units of linear, capacity, and weight measures by converting within the customary system. ----- 137-142
37. Change units of linear, capacity, and weight measures by converting within the metric system. -------- 137-142
38. Convert linear customary units of measure of inches, feet, and yards or metric units of measure of millimeters, centimeters, meters, and kilometers. -- 137-142
39. Convert measures of customary capacity units of quarts and gallons or metric capacity units of milliliters and liters. -- 137-142
40. Convert measures of customary weight units of ounces and pounds or metric weight units of grams and kilograms. -- 137-142

PROBABILITY AND STATISTICS

1. Make a prediction for an outcome. -- 143-147
2. Determine probability using experiments. -- 143-147
3. Use a graph or a table to organize data and determine the range for a set of data. ------------------ 143-147
4. Identify the mean, the median, and the mode for a set of data. -------------------------------------- 143-147
5. Given a frequency table, determine the number of cases on which a score distribution is based. ------- 143-147
6. Given a frequency table, name the number of scores at or below a given score. ----------------------- 143-147
7. Solve word problems requiring the interpretation of a picto-, bar, line, or circle graph and the use of information contained in the graph. --- 148-152
8. Read and construct tables or graphs used to organize data. --- 148-152
9. Find the probability of single and multiple outcomes. -- 143-147
10. Determine the probability of independent and dependent events. ------------------------------------ 143-147
11. Make a prediction for an outcome. --- 143-147

GENERAL MATHEMATICS

Table of Contents

PAGE

RATIO AND PROPORTION

1. Solve word problems requiring the calculation of the ratio of two numbers. ---------- 153-154
2. Solve a proportion in which the unknown is a numerator in one ratio and a denominator in the other ratio. ---------- 153-154
3. Solve a proportion in which a denominator is unknown. ---------- 153-154
4. Solve word problems relating to scale drawings using whole numbers. ---------- 155-156
5. Use scale drawings and proportions to determine actual lengths. ---------- 155-156
6. Write decimals as percents and percents as decimals. ---------- 157-158
7. Demonstrate an understanding that a percent is actually the ratio of a number to 100. ---------- 157-158

CONSUMER MATHEMATICS

1. Solve word problems requiring the calculation of: the final cost when the discount, or sales tax rate, and item costs are given; the amount of discount, tax, commission, or downpayment when the rate and total cost are given; or the ratio of two whole numbers. ---------- 159
2. Solve word problems requiring: calculation of the total purchase price and determination of the correct change; determination of change when the purchase price and the amount tendered are given; recognition of equivalent amounts using different combinations of coins and currency. ---------- 159
3. Solve word problems requiring: the selection of the best of four purchases of the same item; determination of savings when the same item is offered at different prices. ---------- 159
4. Solve word problems requiring: computation of the interest charged on a credit purchase; computation of the price difference between a credit purchase and a cash purchase; comparing cost of cash and credit purchase from the information given in an advertisement. ---------- 159
5. Solve word problems requiring: solution of realistic problems involving time and money; use of information given in chart form. ---------- 159
6. Given a blank check or money order, determine the correct placement of a given item of information. -- 159
7. Given a completed check or money order, determine incorrect placement or error in placement of information. ---------- 159

GENERAL MATHEMATICS

Table of Contents

PAGE

CONSUMER MATHEMATICS (Cont'd)

8. Solve word problems requiring: computation of net pay when salary and deductions are given; computation of net pay when amount of deductions, the rate of pay, and the number of hours worked are given. ---------- 159
9. Solve word problems for: miles per gallon when given distance in miles and gallons of fuel used; time when given distance traveled and average rate of speed; rate when given distance traveled and time. -- 159

ORIGINAL SOURCES AND STUDENT OUTCOMES ---------- 162-202

GENERAL MATHEMATICS

AREA: WHOLE NUMBERS

OUTCOMES:
1. Solve addition problems with no more than five addends.
2. Solve addition problems with addends up to five digits.
3. Solve addition problems with or without regrouping.
4. Compute sums presented in horizontal or vertical format.
5. Subtract numbers with subtrahends and minuends of up to five digits with or without regrouping.
6. Solve subtraction problems presented in horizontal or vertical format.
7. Multiply one-, two-, and three-digit factors.
8. Solve multiplication problems presented in horizontal or vertical format.
9. Solve division problems with dividends up to five digits with or without remainders.
10. Solve division problems presented symbolically by: $a \div b$ or $b\overline{)a}$.
11. Interpret and solve a division problem stated in fractional format, i.e., $\frac{a}{b}$.

AREA: DECIMALS

OUTCOMES:
4. Add two or more decimal numbers.
5. Add decimal numbers and whole numbers.
6. Add decimal numbers and mixed decimals.
7. Add whole numbers and mixed decimals.
8. Add two or more mixed decimals.
9. Add sums involving up to three unlike decimal addends.
10. Add decimal numbers to two places with regrouping and with no more than five digits to the left of the decimal.
11. Add sums involving money values.
12. Compute sums presented in horizontal or vertical format.
13. Subtract one decimal number from another.
14. Subtract a whole number from a decimal number and vice versa.
15. Subtract a decimal number from a mixed decimal.
16. Subtract one mixed decimal from another.
17. Subtract decimal numbers to two places with regrouping.
18. Subtract money values.
19. Subtract decimal numbers with no more than five digits to the left of the decimal.
20. Solve subtraction problems presented in horizontal or vertical format.

OUTCOMES: 4 - 37 (Cont'd)

21. Multiply: whole numbers and decimal numbers; whole numbers and mixed decimals; a decimal number by another decimal number; decimal numbers and mixed decimals; and a mixed decimal by another mixed decimal.
22. Multiply a decimal by a power of ten by relocating the decimal point.
23. Multiply hundredths times tenths.
24. Multiply hundredths times hundredths.
25. Multiply money values.
26. Multiply decimals with no more than three digits to the left of the decimal.
27. Multiply decimal numbers presented in horizontal or vertical format.
28. Divide a decimal number or a mixed decimal by a whole number, correctly placing the decimal point in the quotient.
29. Express the quotient of two whole numbers as a decimal number.
30. Divide by powers of ten by relocating the decimal point.
31 Rewrite and solve a division problem with a decimal number or mixed decimal divisor as an equivalent problem with a whole number divisor.
32. Divide hundredths by tenths.
33. Divide tenths by whole numbers of not more than two digits.
34. Divide decimal numbers with no more than two digits to the left of the decimal in the divisor.
35. Divide decimal numbers with dividends not to exceed five digits.
36. Select a whole percent up to 200%, equivalent to a given decimal number, up to two places.
37. Select a decimal number up to two places, equivalent to a given percent, up to 200%.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Have a knowledge of place value and the four basic operations.

Step to Success

1. The ability to compute automatically the basic facts and to regroup according to place value will help the students with other mathematical skills.

ACTIVITY 1 - AREA: WHOLE NUMBERS - OUTCOMES 1, 5, 7, and 9
AREA: DECIMALS - OUTCOMES 4, 13, 21, and 29

Review the basic facts without using paper and pencil. Use examples such as:

	Teacher Says	Students Think
1.	5 + 3	(8)
	add 4	(12)
	add 60	(72)
2.	add 20 to 12	(32)
	divide by 4	(8)
	multiply by 2	(16)
	subtract 1.5	(14.5)

ACTIVITY 2 - AREA: WHOLE NUMBERS - OUTCOMES 1, 2, 3, and 4
AREA: DECIMALS - OUTCOMES 5 - 12

Give students addition problems. Make the problems progressively more difficult.

Example 1: $6 + 7 + 3 + 8 + 9 =$

Example 2: $\begin{array}{r} 3.54 \\ \underline{+2.35} \end{array}$

Example 3: $\begin{array}{r} 6348 \\ \underline{+\ 536.5} \end{array}$

Example 4: $\begin{array}{r} 78079 \\ \underline{+\ 6784} \end{array}$

Example 5: $4.84 + 3.002 + 12.1 =$

Example 6: \$12.53 + \$13.05 =

ACTIVITY 3 - AREA: WHOLE NUMBERS - OUTCOMES 5 and 6
AREA: DECIMALS - OUTCOMES 14 - 20

After reviewing basic subtraction facts, give students problems starting at the lowest level of difficulty and progress to more difficult problems.

Example 1: 18 - 7 =

Example 2: 25 - 1.3 =

Example 3:
$$\begin{array}{r} \$65.10 \\ \underline{-\$32.60} \end{array}$$

Example 4:
$$\begin{array}{r} 75{,}346 \\ \underline{-\ 8{,}738} \end{array}$$

Example 5:
$$\begin{array}{r} 16.078 \\ \underline{-\ 9.384} \end{array}$$

ACTIVITY 4 - AREA: WHOLE NUMBERS - OUTCOMES 7 and 8
AREA: DECIMALS - OUTCOMES 22 - 31

Review the basic multiplication facts. If appropriate, play a game with a tennis ball. Toss the ball to a student as you call out a fact such as 6 x 8. The answer (48) must be given as the student catches the ball.

Give students problems to multiply.

Example 1:
$$\begin{array}{r} 286 \\ \underline{\times\quad 5} \end{array}$$

Example 2:
$$\begin{array}{r} 286 \\ \underline{\times\ .47} \end{array}$$

Example 3:
$$\begin{array}{r} 76.4 \\ \underline{\times\ \ 823} \end{array}$$

ACTIVITY 4 - AREA: WHOLE NUMBERS - OUTCOMES 7 and 8
AREA: DECIMALS - OUTCOMES 22 - 31
(Cont´d)

Example 4: \$704 x 48 =

Example 5: 0.68 x 5.0 =

ACTIVITY 5 - AREA: WHOLE NUMBERS - OUTCOMES 9, 10, and 11
AREA: DECIMALS - OUTCOMES 29 - 37

Review division problems with one-digit divisors.

Example 1: $6\overline{)37.8}$

Example 2: $36\overline{)4}$

Example 3: $0.48 \div 0.4$

Review dividing with two- and three-digit divisors.

Example 1: $30\overline{)6780}$

Example 2: $0.16\overline{)4568}$

Example 3: $38{,}462 \div 451 =$

GENERAL MATHEMATICS

AREA: WHOLE NUMBERS

OUTCOMES:

1. Solve addition problems with no more than five addends.
2. Solve addition problems with addends up to five digits.
3. Solve addition problems with or without regrouping.
4. Compute sums presented in horizontal or vertical format.
5. Subtract numbers with subtrahends and minuends up to five digits with or without regrouping.
7. Multiply one-, two-, and three-digit factors.
9. Solve division problems with dividends up to five digits with or without remainders.
17. Read powers of numbers and find the products they indicate.
20. Identify the square roots of perfect squares.

NOTES FOR THE TEACHER

The activities included in this lesson are motivational activities that provide practice in computation with whole numbers.

ACTIVITY 1 - OUTCOMES 1, 2, 3, 4, 5, 7, and 9

Put the following activity on the chalkboard or on worksheets.

A. Choose an odd number less than 210 and write it: ____.

B. a. Divide the number by 2. Write the quotient: ____. Write the remainder: □ .

b. Divide the number by 3. Write the quotient: ____. Write the remainder: ○ .

c. Divide the number by 5. Write the quotient: ____. Write the remainder: △ .

d. Divide the number by 7. Write the quotient: ____. Write the remainder: ⬡.

ACTIVITY 1 - OUTCOMES 1, 2, 3, 4, 5, 7, and 9 (Cont'd)

C. Compute.

□ x 105 = ____

○ x 70 = ____

△ x 126 = ____

⬡ x 120 = ____

Sum = ⏢

D. Subtract 210 from the ⏢ repeatedly until the result is less than 210.

E. Compare the answer with the number with which you started.

F. Repeat the steps using another number.

ACTIVITY 2 - OUTCOMES 1, 2, 3, 5, 7, 9, 17, and 20

A. Write the year of your birth.

B. Use the digits in the year of your birth exactly one time and any sign of operation to generate the numbers from 1 to 100, or as far as you can.

For example: If you use the year 1986

$1 = \frac{8 - 6}{\sqrt{9} - 1}$, $2 = (\sqrt{9} + 6 + 1) - 8$, $3 = \frac{1 + 8}{9 - 6}$ ... $27 = 9 \times (18/6)$ ... .

There are many ways to express each number.

GENERAL MATHEMATICS

AREA: WHOLE NUMBERS: ESTIMATION AND APPROXIMATION

OUTCOMES:
12. Interpret place value in numerals of up to seven digits and round numbers.
13. Estimate sums, differences, products, and quotients of whole numbers.
14. Estimate a sum up to five addends.
15. Round whole numbers to tens, hundreds, and thousands.
16. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.

AREA: DECIMALS: ESTIMATION AND APPROXIMATION

OUTCOMES:
41. Identify place value in decimal numbers and mixed decimals up to six digits.
42. Round decimal numbers and mixed decimals to a specified place value.
43. Estimate to the nearest whole number sums, differences, products, and quotients involving decimal numbers or mixed decimals.
44. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.
45. Estimate to the nearest ten cents or dollar.
46. Round to the nearest dollar.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Compare whole numbers and decimals.
2. Read and write whole numbers and decimals.
3. Add, subtract, multiply, and divide whole numbers and decimals.

NOTES FOR THE TEACHER (Cont'd)

Steps to Success

1. To round whole numbers, locate the place in the digit that is to be rounded. Write all digits to the left of the required place as they appear. If the digit to the right of the required place is five or more, increase the digit in the required place by 1. If it is less than five, write the same digit as given. Then write a zero in place of each digit to the right of the required place.

2. To round decimals the same procedure will be used as in rounding whole numbers. The exception is that all digits to the right of the required place will be dropped.

ACTIVITY 1 - AREA: WHOLE NUMBERS - OUTCOMES 12, 13, 14, 15, and 16
AREA: DECIMALS - OUTCOMES 41, 42, 43, 44, 45, and 46

Ask students to define a ball park figure.

Explain that a ball park figure is a broad estimate and not an accurate count.

Sometimes it is helpful to know an approximate value of a number rather than the exact value.

Example 1: How many students attended a performance by the drama club?

Example 2: Ask students how much money will be needed to make a trip.

Many times a group of numbers is given but only an estimate of their sums, differences, products, or quotients is needed.

Example 1: Is a $20 bill enough money to purchase items which cost $3.89, $5.28, and $10.11?

ACTIVITY 1 - AREA: WHOLE NUMBERS - OUTCOMES 12, 13, 14, 15, and 16
AREA: DECIMALS - OUTCOMES 41, 42, 43, 44, 45, and 46
(Cont´d)

Examples such as the following will provide students with needed practice in rounding numbers.

Example 1: Given the following number:

654,321.784321

Round the above number

1. to the nearest 10.
2. to the nearest ten thousand.
3. to the nearest hundred thousand.
4. to the nearest hundredth.
5. to the nearest hundred-thousandth.

Example 2: Given the following two numbers:

$52246.05 and $4,025.80

Estimate their

1. sum,
2. difference,
3. product, and
4. quotient.

ACTIVITY 1 - AREA: WHOLE NUMBERS - OUTCOMES 12, 13, 14, 15, and 16
AREA: DECIMALS - OUTCOMES 41, 42, 43, 44, 45, and 46
(Cont'd)

If the two numbers are rounded to the thousands place, the estimated sum, difference, product, and quotient will be as follows:

$$\begin{array}{r} \$52,000 \\ +\ \ 4,000 \\ \hline \$56,000 \end{array} \qquad \begin{array}{r} \$52,000 \\ -\ \ 4,000 \\ \hline \$48,000 \end{array} \qquad \begin{array}{r} \$52,000 \\ \times\ \ \ \ 4,000 \\ \hline \$208,000,000 \end{array} \qquad \begin{array}{r} \$13. \\ \$4,000\overline{)\$52,000.} \\ 40,00\ \ \\ \hline 12,000 \\ 12,000 \\ \hline \end{array}$$

Example 3: There are 256 males and 198 females in the freshman class, 301 males and 358 females in the sophomore class, 285 males and 225 females in the junior class, and 379 males and 415 females in the senior class. To the nearest hundred, how many males and how many females are in the school?

	MALE	FEMALE
Freshmen	300	200
Sophomores	300	400
Juniors	300	200
Seniors	400	400
TOTAL	1300	1200

58

59

AREA: WHOLE NUMBERS: POWERS AND SQUARE ROOTS

OUTCOMES:
17. Read powers of numbers and find the products they indicate.
18. Identify the expanded form of a number expressed in exponential form.
19. Subtract a number less than ten from a number expressed in exponential form.
20. Identify the square roots of perfect squares.
21. Find square roots using a table of square roots.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Be able to perform the four basic operations.

ACTIVITY 1 - OUTCOMES 17, 18, and 19

In mathematics short cuts are often used. For example, it is easy enough to write 2 x 2 x 2. This is understood to mean multiply 2 three times; but, to write 2 multiplied one hundred times would take a very long time.

There is a short method for writing numbers that are going to be multiplied again and again.

$2^1 = 2$ Read: 2 to the first power

$2^2 = 2 \times 2 = 4$ 2 to the second power

$2^3 = 2 \times 2 \times 2 = 8$ 2 to the third power

$2^4 = 2 \times 2 \times 2 \times 2 = 16$ 2 to the fourth power

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ 2 to the fifth power

ACTIVITY 1 - OUTCOMES 17, 18, and 19 (Cont'd)

1. Have students read expressions including the powers.

   Example: 3^2, 10^3, 5^4

2. Have students write numbers expressed in exponential form in their expanded forms.

   Examples: 3^4 would be written as $3 \times 3 \times 3 \times 3$

   7^5 would be written as $7 \times 7 \times 7 \times 7 \times 7$

   12^3 would be written as $12 \times 12 \times 12$

3. Have students simplify expressions containing exponential forms.

   Examples: $12^2 - 8 =$

   $10^3 - 9 =$

   $3^2 - 2 =$

ACTIVITY 2 - OUTCOMES 20 and 21

Ask students to describe a perfect square.

Describe a perfect square number. A perfect square is a number that has been multiplied by itself.

Examples:

$$\left.\begin{array}{l} 1^2 = 1 \times 1 = 1 \\ 2^2 = 2 \times 2 = 4 \\ 3^2 = 3 \times 3 = 9 \\ 4^2 = 4 \times 4 = 16 \\ 5^2 = 5 \times 5 = 25 \end{array}\right\} \text{These are perfect squares.}$$

ACTIVITY 2 - OUTCOMES 20 and 21 (Cont'd)

Ask students to look at perfect squares in another way.

Provide students with perfect squares. Ask students to decide what number was squared.

Examples: $36 = 6 \times 6 = 6^2$

$81 = 9 \times 9 = 9^2$

$144 = 12 \times 12 = 12^2$

$\sqrt{}$ is the symbol for square root; therefore, 6 is the square root of 36 and is written $\sqrt{36} = 6$; 9 is the square root of 81 and is written $\sqrt{81} = 9$; 12 is the square root of 144 and is written $\sqrt{144} = 12$.

Of course, all numbers are not perfect squares. Square root tables can be used with numbers that are not perfect squares.

AREA: DECIMALS: COMPARE AND ORDER

OUTCOMES:
1. Identify place value in decimal numbers and mixed decimals up to six digits.
2. Compare decimal numbers between zero and one to thousandths.
3. Select from the options a decimal less than or greater than a given decimal.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Read whole numbers.
2. Understand the meaning of less than and greater than.

Steps to Success

1. The student will need to understand that decimal numbers may include a fractional part of a whole. The digit(s) to the left of the decimal comprise the whole number. The digit(s) to the right of the decimal comprise the fractional part of the number.

2. To read a decimal number, read the whole number and the word "and." Then read the number to the right of the decimal as if it were a whole number followed by the name of the last decimal place in the number.

3. Zeros are used to hold places when writing decimals.

   Example: One hundred twenty five ten-thousandths = .0125

ACTIVITY 1 - OUTCOMES 1, 2, and 3

tens	ones		tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
4	3	.	5	9	2	5	4	8

This number contains what we call a decimal fraction or simply a decimal. The decimal is separated from the whole number by a decimal point. The complete number above is called a mixed decimal. It is read "forty three and five hundred ninety-two thousandths five hundred forty-eight millionths."

Remind students to say "and" at the decimal point when reading mixed decimals.

Provide students problems for practice.

Example 1: State the place the 5 holds in each number.

1. 52.689
2. 1.5378
3. .0050
4. .108325

Example 2: Which decimal number is greater: .563 or .54?

Solution: When comparing decimal numbers such as .563 and .54 annex a zero to give the numbers being compared the same number of digits to the right of the decimal.

.563
.540

ACTIVITY 1 - OUTCOMES 1, 2, and 3 (Cont'd)

Read the numbers as if no decimals were there.

five hundred sixty-three
five hundred forty

Therefore, .563 > .54

AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER

OUTCOMES:
1. **Identify the fraction that is equivalent to a given common fraction.**
2. **Identify the common fraction that is expressed in simplest form.**
3. **Rename a common fraction to simplest form.**
4. **Rename a mixed number as an improper fraction.**

AREA: FRACTIONAL NUMBERS: BASIC OPERATION

OUTCOMES:
33. **Change a mixed number to an improper fraction.**
34. **Change an improper fraction to a mixed number.**

NOTES FOR THE TEACHER

Prerequisites for Students

1. Recognize that a common fraction is part of a whole.
2. Add, subtract, multiply, and divide whole numbers and fractions.
3. Demonstrate what the terms numerator and denominator mean.
4. Identify proper and improper fractions and mixed numbers.

Steps to Success

1. Outcomes 1, 2, and 3 depend upon students understanding that for any common fraction there is an infinite number of equivalent fractions.

   1/2 = 2/4 = 3/6 = 4/8 ... = 10/20 ... = 500/1000 ...

   Therefore, Activity 1 deals with the generation of equivalent fractions.

   A fraction is defined as being in it simplest form when there is no number (other than 1) that can be divided equally into the numerator and denominator.

NOTES FOR THE TEACHER (Cont'd)

2. Use of transparencies and an overhead projector are recommended for Activity 1. Overlapping transparencies are especially helpful in showing equivalent fractions.

3. Another important awareness in comparing and ordering fractions is the understanding that equivalent fractions are generated either by multiplying both numerator and denominator by the same number or by dividing the numerator and denominator by the same number.

   1. Multiplying by the same number

$$\frac{10}{20} = \frac{10 \times 2}{20 \times 2} = \frac{20}{40}$$

$$= \frac{10 \times 3}{10 \times 3} = \frac{30}{60}$$

$$= \frac{10 \times 5}{20 \times 5} = \frac{50}{100}$$

   2. Dividing by the same number

$$\frac{10}{20} = \frac{10 \div 2}{20 \div 2} = \frac{5}{10}$$

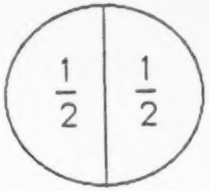
$$= \frac{10 \div 5}{20 \div 5} = \frac{2}{4}$$

$$= \frac{10 \div 10}{20 \div 10} = \frac{1}{2}$$

Activity 2 deals with this awareness.

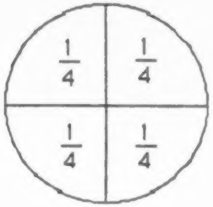
ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOME 1

Draw a circle on a transparency. Divide it into 2 equal parts.



Point out that each part has a name.
Each part can be thought of as "one of two equal parts" or 1/2.
Label each part 1/2.

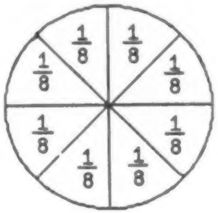
Draw a circle of the same size on a second transparency. Divide this circle into 4 equal parts.



Point out that each part has a name.
Each part can be thought of as "one of four equal parts" or 1/4.
Label each part 1/4.
Overlap the second circle with the first circle to illustrate:

1/2 = 1/4 + 1/4
1/2 = 2/4

Draw a circle of the same size on a third transparency. Divide this circle into 8 equal parts.



Point out that each part has a name. (1/8)
Overlap the third circle with the first and second circles to illustrate:

1/2 = 1/4 + 1/4 or 1/2 = 2/4
1/2 = 1/8 + 1/8 + 1/8 + 1/8 or 1/2 = 4/8
Therefore, 1/2 = 2/4 = 4/8

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOME 1 (Cont´d)

Shift from the use of concrete drawings to a series of leading questions similar to the following.

1. If the same process were continued, how many parts would be shown in the next drawing? (16)
2. What would each part be named? (1/16)
3. How many sixteenths would be equal to 1/2? (8/16)

Record the findings on the chalkboard as follows:

1/2 = 2/4 = 4/8 = 8/16

Point out that this "family" of equivalent fractions is infinite.
Have students complete examples similar to the following and to verbalize how they found each solution.

Example: 1/2 = 2/4 = 4/8 = 8/16 = ... = /24 = ... 50/ = ... =

Solution: 1/2 = 12/24 because both the numerator and the denominator were multiplied by 12.

$$\frac{1}{2} = \frac{1 \times 12}{2 \times 12} = \frac{12}{24}$$

1/2 = 50/100 because both the numerator and the denominator were multiplied by 50.

$$\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \frac{50}{100}$$

Expand the examples to include families of equivalent fractions for a variety of common fractions. Emphasize that equivalent fractions are generated by multiplying both numerator and denominator by the same number.

Example: 1/3 = 2/6 = ... = 5/ = ... = 10/ = ... = /39 = ...

5/7 = ... = 15/ = ... = /42 = ... =

77

78

ACTIVITY 2 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOMES 1, 2, and 3

Start with an example that was worked out in Activity 1.

$1/2 = 2/4 = 3/6 = 4/8 = 5/10 \ldots = 25/50 = \ldots = 30/60 = \ldots = 300/600$

Point out that students may be given a fraction and asked to generate equivalent fractions in which the numerators and denominators are larger than the given fraction.

Example: Ask students to find members of the family of equivalent fractions that would fall between 30/60 and 300/600.

Solution: $30/60 = 60/120 = 90/180 = 120/240 = \ldots$

Emphasize that these equivalent fractions were generated by multiplying the numerator and denominator of the given fraction by the same number.

$$\frac{30}{60} = \frac{30 \times 2}{60 \times 2} = \frac{60}{120} = \frac{30 \times 3}{60 \times 3} = \frac{90}{180} \ldots$$

Point out that students may be given a fraction and asked to find equivalent fractions in which the numerators and denominators are smaller than the given fraction.

Example: Ask students to find equivalent fractions for 30/60 in which the numerators and denominators are smaller than 30.

Solution: $30/60 = 15/30 = 6/12 = 5/10 = 1/2$

Emphasize that these equivalent fractions were generated by dividing the numerator and denominator of the given fraction by the same number.

$$\frac{30}{60} = \frac{30 \div 2}{60 \div 2} = \frac{15}{30}$$

$$\frac{30}{60} = \frac{30 \div 5}{60 \div 5} = \frac{6}{12}$$

$$\frac{30}{60} = \frac{30 \div 6}{60 \div 6} = \frac{5}{10}$$

ACTIVITY 2 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOMES 1, 2, and 3

$$\frac{30}{60} = \frac{30 \div 15}{60 \div 15} = \frac{2}{4}$$

$$\frac{30}{60} = \frac{30 \div 30}{60 \div 30} = \frac{1}{2}$$

Tell students that 1/2 is 30/60 reduced to its simplest form. Explain that a fraction is in its simplest form when the numerator (1) and the denominator (2) cannot be divided equally by a number other than 1.

Give students a variety of examples to reduce to simplest terms.

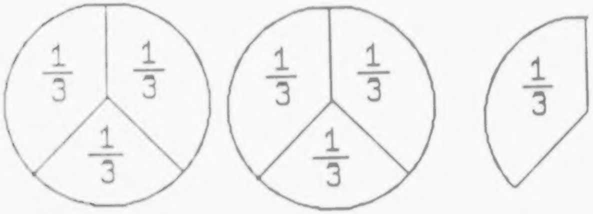
Examples: 9/12 =

16/24 =

8/20 =

ACTIVITY 3 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOME 4
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 33 and 34

Start by defining a mixed number as a whole number and a fraction (2 1/3). Begin with a mixed number that is easy to illustrate through a drawing. Illustrate a mixed number similar to 2 1/3.



Point out and explain the following.

2 1/3 = 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3
2 1/3 = 3/3 + 3/3 + 1/3
2 1/3 = 2(3/3) + 1/3
2 1/3 = 7/3

Explain to students that the way of changing mixed numbers to improper fractions can be illustrated through drawings, as long as the numbers remain small.

ACTIVITY 3 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOME 4
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 33 and 34
(Cont´d)

Give an example that involves larger numbers and have students work out a solution that will apply to changing all mixed number to improper fractions.

Example: Change 105 2/5 to an improper fraction.

Solution: 1. How many fifths would be in 105? (105 x 5 or 525)

2. How many additional fifths? (2)
How many fifths in all? (527)
Write as a fraction. (527/5)

Work several examples in this manner until students understand what is involved in changing mixed numbers to improper fractions. Then, show that this process may be speeded up by use of the more familiar procedure.

Example: Change 105 2/5 to an improper fraction.

Solution: 1. Multiply the whole number (105) by the denominator of the fraction (5).

$$105\frac{2}{5} = \frac{525}{}$$

2. Add the numerator of the fraction (2) to the product found in Step 1.

$$105\frac{2}{5} = \frac{525 + 2}{}$$

3. Write the sum found in Step 2 (527) over the denominator of the fraction.

$$105\frac{2}{5} = \frac{527}{5}$$

ACTIVITY 3 - AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER - OUTCOME 4
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 33 and 34
(Cont´d)

Students should be encouraged to use the procedure that makes the most sense to them. Provide a variety of examples for practice. Be sure to explain that improper fractions can be changed to mixed numbers through simple division.

Example: Change 10/7 to a mixed number.

Solution: 10/7 can be read as "10 divided by 7".

$10 \div 7 = 1\ 3/7$

Emphasize that improper fractions are considered to be in simplest form when they are converted to mixed numbers. In examples provided for practice, include both types of problems.

Type 1: Change the following to improper fractions.

1 7/8 =
30 2/5=

Type 2: Simplify the following:

10/6 =
31/30 =

AREA: FRACTIONAL NUMBERS: COMPARE AND ORDER

OUTCOMES:
5. Find the least common denominator for three common fractions.
6. Identify the greatest of four common fractions with unlike denominators.
7. Order three common fractions with unlike denominators from least to greatest.
8. Identify the proper fractions that are less than or greater than a given proper fraction.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Change fractions to equivalent fractions.
2. Order whole numbers.
3. Find the least common multiple for whole numbers.

Steps to Success

1. Outcomes 6, 7, and 8 depend upon the ability to identify a least common denominator for fractions (Outcome 5). Therefore, Activity 1 serves as a review for finding least common denominators. This topic is covered thoroughly also in the *Mathematics Curriculum Guides*: *Grades 7-8*, Area: Fractions and Decimals, Outcome 1.

2. Outcomes 6, 7, and 8 deal primarily with three different ways of asking questions about comparing and ordering fractions. These formats are:

   Outcome 6: Which fraction listed below has the greatest value?

   A. 5/9
   B. 1/2
   C. 3/5
   D. 7/8

NOTES FOR THE TEACHER (Cont´d)

Outcome 7: Order from least to greatest:

5/9, 3/5, 7/8

Outcome 8: Which fraction listed below is greater than 3/4?

A. 5/9
B. 1/2
C. 3/5
D. 7/8

ACTIVITY 1 - OUTCOME 5

In the previous three activities (Outcomes 1, 2, 3, and 4) students were shown how to find equivalent fractions. The ability to find equivalent fractions is the skill needed to find the lease common denominator for fractions with unlike denominators.

Use an everyday example such as pieces of fruit. Point out that the sum of 1 banana, 2 apples, and 3 oranges is 1 banana, 2 apples, and 3 oranges. Only by changing these different names to a common name is it possible to find a sum.

1 banana = 1 piece of fruit
2 apples = 2 pieces of fruit
3 oranges = 3 pieces of fruit

The sum, therefore, is 6 pieces of fruit.

Relate this example to finding the sum of fractions with unlike denominators.

ACTIVITY 1 - OUTCOME 5 (Cont´d)

Example: Add 1/2, 1/3, and 1/5

Solution: 1. Name the multiples of each number in the denominator.

Multiples of 2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, $\boxed{30}$
(Stop when a multiple is found that is also a multiple of the other two numbers in the denominators.)

Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, $\boxed{30}$

Multiples of 5 = 5, 10, 15, 20, 25, $\boxed{30}$

2. Change each given fraction into an equivalent fraction with a denominator of the least common multiple identified in Step 1. (30)

1/2 = 15/30
1/3 = 10/30
1/5 = 6/30

3. Add the new fractions that have a common denominator.

1/2 = 15/30
1/3 = 10/30
1/5 = + 6/30
31/30 or 1 1/30

Be sure that students understand that the LEAST COMMON DENOMINATOR for given fractions (1/2, 1/3, 1/5) is the least common multiple for the denominators of the given fractions (2, 3, and 5).

ACTIVITY 2 - OUTCOME 5

Students may need ample practice in finding least common denominators. Sufficient practice will increase the speed with which common denominators are found. For such practice, play the game "Make Us the Same". Use a deck of flash cards. On each card, write 2 or 3 unlike fractions.

2/5 , 3/10

4/7 , 5/9

1/2 , 1/3 , 2/7

Tell students to think of the smallest number that each of the denominators will divide into evenly. Call on individual students or divide the class into teams. If the team approach is chosen, a point can be scored by each team for each correct answer.

By writing the correct answer on the back of each card, the game can be adapted for independent use. The game can be expanded to have students convert the given fractions to fractions that have a common denominator.

Example: 2/5, 3/10 Common denominator = 10 Conversion: 2/5 = 4/10 3/10 = 3/10

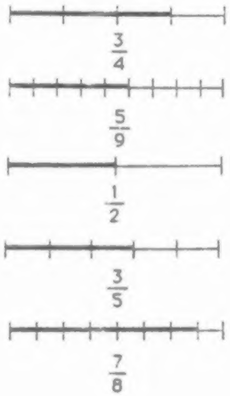
ACTIVITY 3 - OUTCOME 8

Have the students draw a line segment on their papers. Lengths may vary. Then have students divide the line into the correct number of units for the fraction given in the example. Under this segment, draw other line segments equal in length to the fractions in which the given fraction is being compared. Be sure to choose an example that can be easily illustrated.

Example: Which fractions in the following group are less than 3/4?

5/9, 1/2, 3/5, 7/8

Solution: By shading the various segments, it can be seen that 5/9, 1/2, and 3/5 are less than 3/4.



ACTIVITY 3 - OUTCOMES 6, 7, and 8 (Cont'd)

Explain that it is time-consuming to solve problems of this nature by making illustrations. Explain that if all fractions have the same denominator, then comparisons can be made easily by comparing the numerators. Provide examples for student practice. Be sure to include the formats listed below.

Example 1: Which fraction listed below has the greatest value?

A. 5/8
B. 1/2
C. 3/5
D. 3/4

Solution:
$5/8 = 25/40$
$1/2 = 20/40$
$3/5 = 24/40$
$3/4 = 30/40$

The answer is 3/4 because $30/40 > 25/40$, $20/40$, and $24/40$.

Example 2: Order from least to greatest:

5/8, 1/2, 3/5

Solution:
$5/8 = 25/40$
$1/2 = 20/40$
$3/5 = 24/40$

1. Order the numerators. $20 < 24 < 25$

2. Therefore, $20/40 < 24/40 < 25/40$

3. Ordered from least to greatest:

   1/2 is least
   3/5 is next
   5/8 is greatest

ACTIVITY 3 - OUTCOMES 6, 7, and 8 (Cont´d)

Example 3: Which fraction listed below is greater than 3/4?

A. 5/8
B. 1/2
C. 3/5
D. 7/8

Solution: 1. Convert given fractions to fractions with the same denominator.

Given fraction: 3/4 = 30/40
Other fractions: 5/8 = 25/40
1/2 = 20/40
3/5 = 24/40
7/8 = 35/40

2. 7/8 is greater than 3/4 because 35/40 > 30/40.

GENERAL MATHEMATICS

AREA: FRACTIONAL NUMBERS: COMPUTATION

OUTCOMES: 9. Add two common fractions with like denominators with renaming.
10. Add three common fractions with unlike denominators.
11. Add two mixed numbers with unlike denominators.
12. Add proper fractions and mixed numbers with unlike denominators.
13. Subtract one mixed number from another with a like denominator with renaming.
14. Subtract a common fraction from a mixed number, each with a different denominator.
15. Subtract proper fractions and mixed numbers with unlike denominators.

AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS

OUTCOMES: 35. Add two or more like or unlike mixed numbers with or without renaming.
36. Add like or unlike mixed numbers and fractions or whole numbers with or without renaming.
37. Subtract a mixed number, a fraction, or a whole number from a like or unlike mixed number with or without renaming.
38. Subtract a mixed number or a proper fraction from a whole number with or without renaming.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Add and subtract whole numbers.
2. Find the lowest common denominator.
3. Reduce fractions.

Steps to Success

1. To add or subtract fractions with like denominators, add or subtract numerators and write the sum over the common denominator. Always reduce the answer to its lowest term.

2. To add or subtract unlike fractions, find their lowest common denominator and change the fractions to equivalent fractions having a common denominator. Then follow the same procedure used in adding and subtracting like fractions. Reduce the answer to its lowest term.

NOTES FOR THE TEACHER (Cont´d)

3. To add or subtract mixed numbers, first add or subtract the fraction and then add or subtract the whole numbers. Reduce the answer to its lowest term.

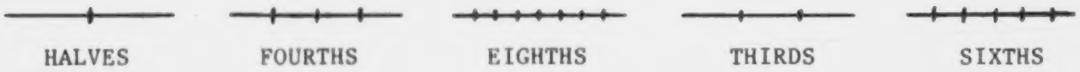
ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 9, 10, 11, 12, 13, 14, and 15
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 35, 36, 37, and 38

This lesson is written as a narrative. Teachers should feel free to teach the lesson using a similar narrative told in their own words. Or, teachers may prefer to ignore the narrative of early man and only use the ideas in the lesson.

Imagine that many years ago when man first started to measure things he probably had only one unit of measure. He may have called this unit of measure an "inc." His communication probably went something like this:

> "My cave is a little more than 60 incs tall." or "I will give you a bit less than two incs of my flint stone."

Of course, he soon found he needed units between each inc to enable him to be more accurate with his measuring tasks. He most likely drew a diagram on the wall of his cave so he could remember the patterns he had created such as:



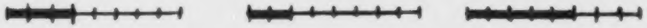
He eventually called these fractions and soon had the need to add and subtract these fractions. He found he had three groups: those with like denominators (like fractions); those with unlike denominators (unlike fractions); and those which contained both whole numbers and fractions (mixed numbers).

Group 1: Like Fractions

It was easy to add to or take away from the fractions when they had the same denominator. Early man could have demonstrated the addition and subtraction of like fractions in the following manner.

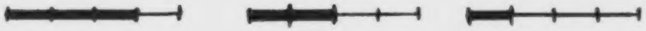
ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 9, 10, 11, 12, 13, 14, and 15
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 35, 36, 37, and 38
(Cont´d)

Example 1:



3/8 + 2/8 = 5/8

Example 2:



3/4 - 2/4 = 1/4

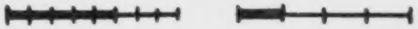
It wasn´t long until early man learned that he could add or subtract the numerators of like fractions and write the sum over the common denominator.

Teachers and/or students should provide other examples of addition and subtraction of like fractions for practice.

Group 2: Unlike Fractions

Early man´s challenge to add and subtract fractions with unlike denominators may have taken longer to solve. But early man was diligent and he succeeded. His success may have occurred in an example similar to the following.

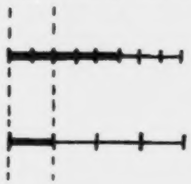
Example 1:



5/8 + 1/4 =

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 9, 10, 11, 12, 13, 14, and 15
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 35, 36, 37, and 38
(Cont'd)

By comparing he noticed that 1/4 was equal to 2/8.



Therefore, the challenge of adding 5/8 and 1/4 was solved as follows.

$$\begin{array}{r} 5/8 = 5/8 \\ \underline{+\ 1/4} = \underline{2/8} \\ 7/8 \end{array}$$

Each time early man had to add or subtract unlike fractions, he renamed one or both of the fractions so that the fractions had a common denominator. Then it was simple to add or subtract because the problem had been changed to one which involved like fractions.

Teachers and/or students should provide other examples similar to the following for practice.

Example 1:
$$\begin{array}{r} 5/12 = 10/24 \\ \underline{+\ 3/8} = \underline{9/24} \\ 19/24 \end{array}$$

Example 2:
$$\begin{array}{r} 7/13 = 14/26 \\ \underline{-\ 3/26} = \underline{3/26} \\ 11/26 \end{array}$$

Example 3:
$$\begin{array}{r} 1/5 = 6/30 \\ 2/6 = 10/30 \\ \underline{+\ 4/15} = \underline{8/30} \\ 24/30 = 4/5 \end{array}$$

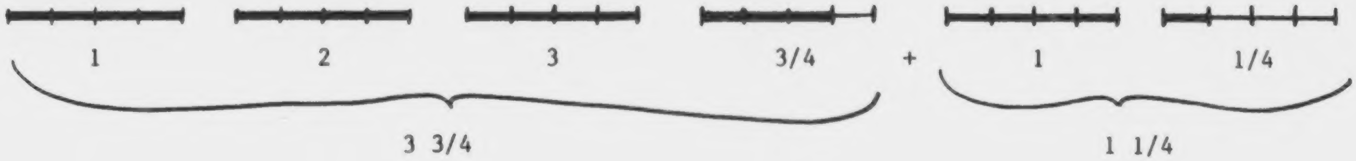
Remind students of the importance of writing the answer in its simplest form.

Group 3: Mixed Numbers

It didn´t take early man long to devise a way to add and subtract mixed numbers. He simply

1. added the whole numbers;
2. added the fractions;
3. found the sum of the whole numbers and fractions; and
4. expressed the answer in its simplest form.

Early man may have demonstrated the addition of mixed numbers as follows.



And, the addition of mixed numbers was written as follows.

$$\begin{array}{rcl} 3\ 3/4 & = & 3 + 3/4 \\ \underline{+\ 1\ 1/4} & = & \underline{+\ 1 + 1/4} \\ & & 4 + 4/4 \ = 4 + 1 = 5 \end{array}$$

The challenge of subtracting mixed numbers was solved in a similar manner.

$$\begin{array}{rcl} 14\ 1/5 & = & 14 + \ 4/20 \\ \underline{-\ 5\ 3/4} & = & \underline{-\ 5 + 15/20} \end{array}$$

Rewrite mixed numbers as whole numbers and fractions with common denominators.

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 9, 10, 11, 12, 13, 14, and 15
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOMES 35, 36, 37, and 38
(Cont´d)

14 + 4/20 = 13 + 24/20
- 5 + 15/20 = - 5 + 15/20

Rewrite again, if necessary, to make the numerator of the minuend fraction larger than the numerator of the subtrahend fraction.

13 + 24/20
- 5 + 15/20
8 + 9/20 = 8 9/20

Subtract the whole number from the whole number and the fraction from the fraction. Express the answer in its simplest form.

Provide other examples similar to the following for practice.

Example 1:
23 1/5
8 5/6
+ 4 1/3

Example 2:
123 5/8
- 1/3

AREA: FRACTIONAL NUMBERS: COMPUTATION

OUTCOMES: 16. Multiply a common fraction by a whole number.
17. Multiply two common fractions with renaming.
18. Multiply two mixed numbers.
19. Multiply two factors of proper fractions and/or mixed numbers.

AREA: FRACTIONAL NUMBERS: BASIC OPERATION

OUTCOME: 39. Multiply mixed numbers with or without renaming.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Reduce common fractions as well as improper fractions.

Steps to Success

1. Like the previous lesson, this lesson is written as a narrative. The purpose of the narrative is to provide students with a sense of how mathematical discoveries may have occurred. Teachers should feel free to teach the lesson using a similar narrative told in their own words. Or, teachers may prefer to ignore the narrative of early man and only use the ideas in the lesson.

2. The multiplication of a whole number by a fraction is introduced first. These problems are solved first because they are easier to approach through concrete examples and illustration. Therefore, if students need to find 1/4 x 8, they can:

   a. Divide 8 into 4 equal groups of 2.
   b. See that 1/4 of 8 = 2.
   c. Rewrite the problem as 1/4 x 8/1 = 2.

d. Observe that the solution is obtained by multiplying numerators (8 x 1) and denominators (4 x 1); expressing the products as a fraction (8/4); and expressing the fraction in its simplest form.

1/4 x 8/1 = 8/4 = 2

Thus, the algorithm for multiplying fractions is introduced.

3. The multiplication of fractions by fractions, fractions by mixed numbers, and mixed numbers by mixed numbers are then introduced and solved by:

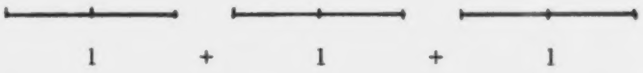
   a. changing all terms to fractions;
   b. multiplying numerators by numerators and denominators by denominators; and
   c. expressing answers in their simplest forms.

4. Cancellation is introduced, without explanation, as a means for having the calculations involve smaller numbers.

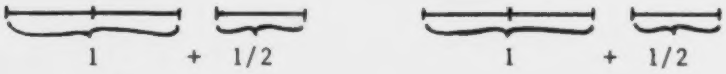
ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 16, 17, 18, and 19
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 39

When early man had the need to find a part of another number, he found it easy as long as he was working with a fractional part of a whole number. One of his first problems may have been similar to the example below.

Example: 1/2 of 3



Early may probably put the first stick in one place and the second stick in another place. Then, he broke the third stick in half. He put half a stick with the first stick and half a stick with the second stick.



ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 16, 17, 18, and 19
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 39
(Cont'd)

In this way, early man divided 3 into two equal parts of 1 1/2. He wrote this in several ways using mathematical symbols.

$3 \div 2 = 1\ 1/2$ and $3/2 = 1\ 1/2$ and $\begin{array}{r} 1\ 1/2 \\ 2\overline{)\ 3\quad} \end{array}$

After saying that 3 had been divided into two equal groups of 1 1/2, early man used other words to describe the relationship between 1 1/2 and 3. He said: "1 1/2 is half of 3." He wrote this in several ways using mathematical symbols.

$1\ 1/2 = 1/2 \times 3$ and $1/2 \times 3 = 1\ 1/2$ and $1/2 \times 3/1 = 3/2$

Early man paid close attention to the statement 1/2 x 3/1 = 3/2. It appeared that the solution to multiplication problems required that the numerator be multiplied (1 x 3) and that the denominators be multiplied (2 x 1).

$1/2 \times 3/1 = 3/2$

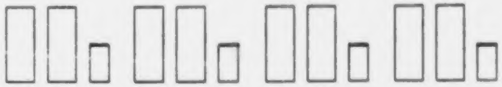
Early man proposed a theory for multiplying fractions:

1. Write the problem so that all terms are fractions.
2. Multiply the numerators.
3. Multiply the denominators.
4. Write the two products as a fraction.
5. Express the answer in simplest form.

Early man tested his theory on other examples involving a fractional part of a whole number. He may have used an example such as the following.

Example: 1/4 of 10 or 1/4 x 10

Solution: First, early man solved the problem using concrete objects. He divided 10 into 4 equal parts.



ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 16, 17, 18, and 19
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 39
(Cont'd)

Each part consisted of 2 1/2 objects.
Then, early man tested his theory.

$1/4 \times 10 =$

1. Write the problem so that all terms are fractions.

$1/4 \times 10/1 =$

2. Multiply the numerators.

$\overset{\frown}{1/4 \times 10}/1 = 10/$

3. Multiply the denominators.

$1/\underset{\smile}{4 \times 10/1} = /4$

4. Write the two products as a fraction.

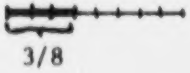
$1/4 \times 10/1 = 10/4$

5. Express the answer in its simplest form.

$1/4 \times 10/1 = 10/4 = 2\ 1/2$

A FRACTIONAL PART OF A FRACTION

It was not long until early man needed to go a step further. He needed to find a fractional part of another fraction. For example, he may have needed to find 1/2 of 3/8.



ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 16, 17, 18, and 19
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 39
(Cont'd)

So early man applied the procedure used to multiply a fraction by a whole number. He multiplied numerators (1 x 3), multiplied denominators (2 x 8) and wrote the products as a fraction (3/16).

1/2 x 3/8 =

1/2 x 3/8 = 3/16

Teachers should provide several examples for practice. Be sure to include examples where canceling keeps the numbers to be multiplied small.

Example without canceling: 5/12 x 6/7 =

5/12 x 6/7 = 30/84 = 5/14

Example with canceling: 5/12 x 6/7 =

$5/\underset{2}{\cancel{12}} \times \overset{1}{\cancel{6}}/7 = 5/14$

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 16, 17, 18, and 19
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 39
(Cont´d)

MULTIPLICATION OF MIXED NUMBERS

With further investigation, early man found that he could multiply mixed numbers simply by rewriting the mixed numbers as improper fractions.

Example: $3\ 2/5 \times 1\ 1/4 =$

Solution: 1. Write the problem so that all terms are fractions.

$3\ 2/5 \times 1\ 1/4 = 17/5 \times 5/4$

2. Cancel if possible, to keep the multiplication simple.

$$3\ 2/5 \times 1\ 1/4 = 17/\underset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}/4$$

3. Multiply numerators and denominators and express the answer in its simplest form.

$$3\ 2/5 \times 1\ 1/4 = 17/\underset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}/4 = 17/4 = 4\ 1/4$$

Teachers should provide a variety of problems for student practice. Be sure to include examples involving fractions multiplied by fractions, whole numbers, and mixed numbers.

AREA: FRACTIONAL NUMBERS: COMPUTATION

OUTCOMES: 20. Divide a whole number by a common fraction.
21. Divide a common fraction by a whole number.
22. Divide a mixed number by a whole number.

AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS

OUTCOME: 40. Divide mixed numbers with or without renaming.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Reduce fractions.
2. Multiply fractions.
3. Demonstrate an understanding of the meaning of invert.

Steps to Success

1. Like the two previous lessons, this lesson is written as a narrative. The purpose of this narrative is to provide students with a sense of how mathematical discoveries may have occurred. Teachers should feel free to teach the lesson using a similar narrative told in their own words. Or, teachers may prefer to ignore the narrative of early man and only use the ideas in the lesson.

2. First, the lesson deals with the division of a fraction by a whole number. These problems are solved first because they resemble the division of whole numbers. Moreover, division of fractions by a whole number makes it possible for students to apply what is already known in order to solve a problem that looks unfamiliar. Specifically:

   a. Students already know how to solve $10 \div 2$.
   They know $10 \div 2 = 5$.
   b. Students also know that 10 can be divided into 2 equal groups of five.

NOTES FOR THE TEACHER (Cont'd)

c. And, since 10 can be divided into 2 equal groups of 5, students also know that 1/2 x 10 = 5. Teachers will need to emphasize that

10 ÷ 2 = 5 also implies 1/2 x 10 = 5

10 ÷ 2 = 5 also implies 10 x 1/2 = 5

3. By extension, the algorithm for dividing problems involving fractions is introduced:

a. Change all terms to fractions.
b. Invert the divisor and change a division problem into an equivalent multiplication problem.
c. Solve in the same manner that multiplication problems involving fractions are solved.

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 20, 21, and 22
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 40

DIVIDE A COMMON FRACTION BY A WHOLE NUMBER

Early man had already seen how to divide a whole number into two equal parts. In the previous lesson, early man divided 3 sticks into 2 equal groups of 1 1/2 sticks. Early man knew also that he could describe this operation in at least two ways.

1. Three divided by two equals one and one-half.

$3 \div 2 = 1\ 1/2$; $\begin{array}{r} 1\ 1/2 \\ 2\overline{)\,3\quad\ } \end{array}$; $3/2 = 1\ 1/2$

2. One-half of three equals one and one-half.

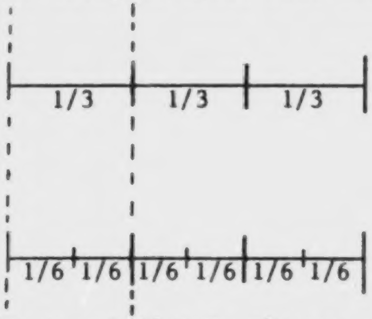
1/2 x 3 = 1 1/2 or 3 x 1/2 = 1 1/2

Thus, early man observed that a division problem (3 ÷ 2 = 1 1/2) could be rewritten as a multiplication problem with the divisor (2) inverted (1/2) and the solution would remain the same (1 1/2).

3 ÷ 2 = 1 1/2 or 3 x 1/2 = 1 1/2

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 20, 21, and 22
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 40
(Cont´d)

This observation proved helpful when early man had the need to divide a fraction by a whole number. For example, perhaps early man needed to know how to divide 1/3 into 2 equal groups. (1/3 - 2). By using diagrams on the walls of his cave, he could find the answer.



A diagram similar to the one above could have shown early man that 1/3 could be divided into 2 equal groups of 1/6. Early man proposed a theory for division problems involving fractions.

Example: 1/3 ÷ 2 =

Theory: 1. Rewrite all terms as fractions.

1/3 ÷ 2/1 =

2. Change the division problem into an equivalent multiplication problem by inverting the divisor.

1/3 ÷ 2/1 = 1/3 x 1/2

3. Solve in the same way that multiplication problems involving fractions are solved.

1/3 ÷ 2/1 = 1/3 x 1/2 = 1/6

Early man´s theory for solving division problems involving fractions proved successful.

ACTIVITY 1 - AREA: FRACTIONAL NUMBERS: COMPUTATION - OUTCOMES 20, 21, and 22
AREA: FRACTIONAL NUMBERS: BASIC OPERATIONS - OUTCOME 40
(Cont'd)

Teachers should provide students with a variety of division problems involving fractions. Be sure to include examples from the following categories:

1. Divide a fraction by a fraction.
2. Divide a whole number by a fraction.
3. Divide a mixed number by a whole number.
4. Divide a mixed number by a fraction.
5. Divide a mixed number by a mixed number.

Teachers should point out that each type of problem can be rewritten so that all terms in the problem are fractions. Then problems can be solved according to the division algorithm. Remind students that cancellation may be used to simplify the calculations. In the early stages, using a step-by-step solution similar to the following may benefit students.

Example: $2\ 3/16 \div 1\ 1/4 =$

Solution: 1. Rewrite all terms as fractions.

$2\ 3/16 \div 1\ 1/4 = 35/16 \div 5/4 =$

2. Change the division problem into an equivalent multiplication problem by inverting the divisor.

$2\ 3/16 \div 1\ 1/4 = 35/16 \times 4/5 =$

3. Simplify through cancellation if possible.

$$2\ 3/16 \div 1\ 1/4 = \overset{7}{\cancel{35}}/\underset{4}{\cancel{16}} \times \overset{1}{\cancel{4}}/\underset{1}{\cancel{5}} =$$

4. Solve in the same way that multiplication problems involving fractions are solved.

$$2\ 3/16 \div 1\ 1/4 = \overset{7}{\cancel{35}}/\underset{4}{\cancel{16}} \times \overset{1}{\cancel{4}}/\underset{1}{\cancel{5}} = 7/4 = 1\ 3/4$$

GENERAL MATHEMATICS

AREA: FRACTIONAL NUMBERS: CONVERSION TO DECIMALS AND PERCENTS

OUTCOMES:

23. Rename a common fraction expressed in thousandths as a decimal fraction.
24. Rename a mixed decimal as a mixed number in simplest form.
25. Rename a decimal fraction as a common fraction in simplest form.
26. Rename a percent as a common fraction in simplest from.
27. Rename a fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 or a mixed number less than ten to a given decimal.
28. Rename a decimal equivalent to a given fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 to a mixed number less than ten.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Read and write whole numbers, decimals, and fractions.
2. Convert fractions to decimals and decimals to fractions.
3. Change a decimal to a percent and a percent to a decimal.

Steps to Success

Many students may already know how to convert decimals to fractions and vice versa. They might know how to convert decimals to percents and vice versa as well. For these students it will be helpful to present practical situations in which all of these skills are needed. This is the approach taken in Activity 1. Other students may need considerable review and practice in making the conversions. Activity 2 provides a format in which students can practice converting fractions to decimals and percents. If students have not mastered conversions, they will need to understand each of the points listed below.

1. The ability to understand that one number can be represented in decimal form, fractional form, or percent form will help the student to convert fractions to decimals and percents.

2. To change a fraction to a decimal, divide the denominator into the numerator.

NOTES FOR THE TEACHER (Cont'd)

3. To rename a decimal in a common fraction, place the decimal numeral in the numerator with a denominator that corresponds to the place value of the last figure in the decimal numeral. Reduce the fraction to its lowest term.

4. To change a mix decimal to a mixed number, make a fraction of the decimal portion and reduce it. Annex the fraction to the whole number.

5. To write a percent as a common fraction, write the percent in decimal form and then change the decimal form to fractional form in its lowest term.

ACTIVITY 1 - OUTCOMES 23, 24, 25, 26, 27, and 28

Suppose a store had a display sign that read:

> \- SALE -
>
> You may have either
>
> 20% off
> or
> 1/4 off
> or
> .22 off
>
> the total purchase
>
> You choose your own DISCOUNT!!

Which of these would you choose if you wanted to save more money?

To solve the dilemma students must be able to compare percents to fractions to decimals. This can easily be done by changing all to percents or all to fractions or all to decimals.

ACTIVITY 1 - OUTCOMES 23, 24, 25, 26, 27, and 28 (Cont'd)

Draw the following chart and discuss its relationship to the problem.

Common Fraction	Decimal Fraction	Percent
______	______	20%
1/4	______	______
______	.22	______

Begin with line 1.

Common Fraction	Decimal Fraction	Percent
______	______	20%

Review the procedure for changing percents to fractions and decimals.

20% = 20/100 or 1/5
20% = .20 or .2

Have students complete line 1.

Common Fraction	Decimal Fraction	Percent
20/100	.20	20%

ACTIVITY 1 - OUTCOMES 23, 24, 25, 26, 27, and 28 (Cont´d)

In the same manner, have students complete lines 2 and 3. Then, direct students to examine the completed table and decide which discount would provide the greatest savings.

Common Fraction	Decimal Fraction	Percent
20/100	.20	20%
25/100	.25	25%
22/100	.22	22%

Be sure that students understand why 1/4 off provides a greater savings than 20% off or .22 off.

ACTIVITY 2 - OUTCOMES 23, 24, 25, 26, 27, and 28

Use a chart similar to the one used in Activity 1. Fill in one item in each line of the chart. Have students complete the other items in each line. Make the items in the chart increasingly complex. Use items similar to the ones listed below.

Fractions	Decimals	Percent
2 1/2		
	3.75	
		33 1/3%
	.045	
		55.6%
1/8		

GENERAL MATHEMATICS

AREA: FRACTIONAL NUMBERS: CONVERSION TO DECIMALS AND PERCENTS

OUTCOME: 28. Rename a decimal equivalent to a given fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 to a mixed number less than ten.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Recognize place value.
2. Line up numbers in columns.
3. Simplify fractions.

Steps to Success

1. The Activity below provides a unique approach to changing decimals to fractions and vice versa. For a different approach see Mathematics Curriculum Guide: Grades 7 and 8, Area: Fractions and Decimal - Outcomes 1 and 5.

2. Encourage students to learn common decimal equivalent for certain fractions. When these equivalents are known, students have a scale with which to judge the reasonableness of other equivalents. Be sure that students know:

1/5 = .20
2/5 = .40
3/5 = .60
4/5 = .80

1/4 = .25
1/2 = .50
3/4 = .75

1/3 = .33 1/3 or .33
2/3 = .66 2/3 or .67

ACTIVITY 1 - OUTCOME 28

The following process can be used to assist students in reading decimal numbers and in writing decimals as fractions or as mixed numbers.

Steps for reading a decimal number:

1. Write the decimal number.	2.368
2. Place a one under the whole number.	2.368 1
3. Fill in all decimal places with zeros.	2.368 1.000
4. Read the number under the given number.	One thousand
5. Read the given number saying the whole number word "two"; the word "and" for the decimal; and the digits after the decimal as a fraction (368/1000).	2 and $\frac{368}{1000}$ or two and three hundred sixty-eight thousandths.

Steps for changing a decimal to a fraction:

1. Write the decimal number.	.52
2. Place a one under the whole number.	0.52 1
3. Fill in all decimal places with zeros.	0.52 1.00
4. Drop the decimal point in the given number and add a division bar.	$\frac{52}{100}$
5. Rewrite in lowest terms.	$\frac{52}{100} = \frac{\cancel{2} \times \cancel{2} \times 13}{\cancel{2} \times \cancel{2} \times 5 \times 5} = \frac{13}{25}$

AREA: FRACTIONAL NUMBERS: ESTIMATION AND APPROXIMATION

OUTCOMES:
29. Make approximations by rounding to whole numbers, proper fractions stated in simplest form and with single digit denominators.
30. Make approximations by rounding to whole numbers, mixed numbers stated in simplest form and with single digit denominators.
31. Estimate sums, differences, products, and quotients involving fractional numbers.
32. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Compare whole numbers and fractions.
2. Read and write fractions.
3. Add, subtract, multiply, and divide fractions.
4. Recognize what the following symbols mean: ≐ (approximately), > (greater than), and < (less than).

Step to Success

1. To round fractions, determine if the fractional part is 1/2 or greater. If it is 1/2 or greater, add one to the whole number. If not, drop the fractional part of the number.

ACTIVITY 1 - OUTCOMES 29, 30, 31, and 32

Often times it is not necessary to have an exact answer. We may need a number that is close to what the exact answer would be in order to make a decision.

ACTIVITY 1 - OUTCOMES 29, 30, 31, and 32 (Cont'd)

The key to rounding fractions is to determine if the fractional part is one half or more. Therefore, begin with examples of common fractions and determine whether each would be a zero (0) or one (1) if rounded.

Example 1: $7/16 \doteq$

Solution: $7/16 \doteq 0$ because $7/16 < 8/16$ or $1/2$.

Example 2: $13/20 \doteq$

Solution: $13/20 \doteq 1$ because $13/20 > 10/20$ or $1/2$.

Extend examples to include rounding mixed numbers to the nearest whole number.

Example 1: $21\ 3/5 \doteq$

Solution: $21\ 3/5 \doteq 22$ because $3/5 > 1/2$ and rounds to 1. $21 + 1 = 22$.

Extend examples to include word problems which require rounding for their solution.

Example 1: Can a pattern calling for 2 1/3 yards of material be used to cut two such items from a piece of material 5 yards long?

Solution: It is not essential to know the actual length required of the two patterns. It is essential to known if there is enough material.

Needed: 2 items of 2 1/3 yards

Round and Add:

$$\begin{array}{r} 2\ 1/3 \\ \underline{+\ 2\ 1/3} \end{array} \begin{array}{c} \doteq \\ \doteq \end{array} \begin{array}{r} 2 \\ \underline{+\ 2} \\ 4 \end{array}$$

Answer: Approximately 4 yards needed. Therefore, the five yards is sufficient.

ACTIVITY 1 - OUTCOMES 29, 30, 31, and 32 (Cont'd)

Example 2: If the pattern had called for two items of 2 7/8 yards each, would 5 yards of material be sufficient?

Solution: Needed: 2 items of 2 7/8 yards

Round and Add:

$$\begin{array}{r c r} 2\ 7/8 & \doteq & 3 \\ \underline{+\ 2\ 7/8} & \doteq & \underline{+\ 3} \\ & & 6 \end{array}$$

Answer: Approximately 6 yards needed. Therefore, the five yards is not sufficient.

Example 3: I have three balls of twine which are 20 1/3 yards, 7 7/8 yards, and 33 3/4 yards long. Do I have enough twine to do a job requiring 60 yards of twine?

Solution: Needed: 20 1/3 yards, 7 7/8 yards, and 33 3/4 yards.

Round and Add:

$$\begin{array}{r c r} 20\ 1/3 & \doteq & 20 \\ 7\ 7/8 & \doteq & 8 \\ \underline{+\ 33\ 3/4} & \doteq & \underline{+\ 34} \\ & & 62 \end{array}$$

Answer: Approximately 62 yards needed. Therefore, the 60 yards is not sufficient.

ACTIVITY 2 - OUTCOMES 29, 30, 31, and 32

Have students solve problems similar to the ones below.

Give the following pairs of numbers, round to estimate their sum, difference, product, and quotient.

Example 1: 4 1/3 and 1 7/8

Solution: Round to nearest whole number.

$4\ 1/3 \doteq 4$
$1\ 7/8 \doteq 2$

Sum: $4 + 2 = 6$
Difference: $4 - 2 = 2$
Product: $4 \times 2 = 8$
Quotient: $4 \div 2 = 2$

Example 2: 20 5/6 and 7 3/4

Solution: Round to the nearest whole number.

$20\ 5/6 \doteq 21$
$7\ 3/4 \doteq 8$

Sum: $21 + 8 = 29$
Difference: $21 - 8 = 13$
Product: $21 \times 8 = 168$
Quotient: $21 \div 8 = 2\ 5/8$

AREA FRACTIONAL NUMBERS: APPLICATIONS

OUTCOMES:

41. Recognize the proper operation needed to solve practical problems involving addition, subtraction, and multiplication of fractions.
42. Solve practical problems involving addition, subtraction, and multiplication of proper fractions and mixed numbers.
43. Solve a one-step division problem in which the answer is expressed as a fraction in simplest form.
44. Solve a rate problem involving a fraction or a mixed number.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Add, subtract, multiply, and divide fractions, decimals, and whole numbers.

Steps to Success

1. Success in solving practical problems depends upon careful reading of the problems and a systematic approach to problem solving.

2. In Activity 1 the strategy of looking for key words that give a clue to the type of operation needed to solve the problem is introduced. It is important to emphasize that the same key words may suggest different operations in different problems. Students must not look at key words in isolation. The key words must be examined in relation to other information given in the problem. For example, the key words "total" and "in all" may be a clue to addition in some problems and multiplication in other problems.

   Example 1: John ate 2 donuts on Monday, 3 donuts on Tuesday, and 4 donuts on Wednesday. How many donuts did he eat in all?

   Solution: Addition is implied because the addends are all different.

NOTES FOR THE TEACHER (Cont'd)

Example 2: John ate 2 donuts on each of the 5 school days this week. How many donuts did he eat in all?

Solution: Multiplication or addition could be used to solve the problem. Multiplication would usually be performed because it is a shortcut for addition problems in which the addends are all the same.

3. In Activity 2, a systematic approach to problem solving is introduced. This five-step process is one way to help students be more orderly in their problem solving. Enough problems should be worked together to make this process become second nature to students.

ACTIVITY 1 - OUTCOMES 41, 42, 43, and 44

Explain to students that working practical problems is similar to going to a scavenger hunt. Solving a word problem requires students to read the problem very carefully and follow directions. Students will benefit also from learning to look for key words which give a clue to the operation that is needed to solve the problem.

Emphasize that there are many ways of giving directions. In math problems, students are directed to perform one of four basic operations: addition, subtraction, multiplication, or division. There are several ways to give directions for each of these operations.

List some of the ways on the chalkboard. Call on students to assist in adding key words to each column.

ADD	SUBTRACT	MULTIPLY	DIVIDE
sum	difference	what part of	quotient
how much in all	how much more	at	ratio
total amount	how much less	in all	each
increased	how much left	total	share equally
gained	decreased	product	divide equally
	reduced		
	remained		

Add to the list as needed. The list may be developed into a chart and posted in the classroom, or students may keep their own individual lists.

To practice looking for key words provide problems that are not to be worked. Instead, the problems are to be read just to determine the operation needed in order to solve them.

ACTIVITY 1 - OUTCOMES 41, 42, 43, and 44 (Cont'd)

Example 1: Gas cost 89 9/10 cents a gallon last month. The price increased by 5 cents this month. What is today's price of gasoline?

Solution: The key word is "increased." The word increased gives the clue that addition is needed to solve this problem.

Example 2: In Mrs. Smith's math class, students' average grade in 1985 was 85 3/4. In 1986 the average grade was 82 1/2. How much did the 1986 class average decrease?

Solution: A clue is found in the key word "decrease." The word decrease suggests that subtraction is needed to solve this problem.

Provide students with a variety of problems containing the key words listed previously. Be sure to include examples in which the same key word(s) may imply different operations in different problems. Emphasize that key words cannot be looked at in isolation. They must be considered in relation to other information provided in the problem. Provide examples similar to the following.

Example 3: Jane has 15 packages to wrap. She plans to put 2 bows on each package. How many bows will she need?

Solution: Each package takes 2 bows.
There are 15 packages.
How many bows implies multiplication because it is assumed that the question is asking for how many bows in all. (2 x 15 = 30)

ACTIVITY 1 - OUTCOMES 41, 42, 43, and 44 (Cont'd)

Example 4: Jane plans to wrap packages for Christmas. It takes 3 5/8 yards of ribbon to wrap each packages. How many packages can be wrapped with 58 yards of ribbon?

Solution: 58 yards of ribbon in all.
3 5/8 yards for each package.
How many packages implies division because the problem is asking how many 3 5/8 yards "go into" 58 yards.

$$(58 \div 3\ 5/8 = \overset{2}{\cancel{58}}/1 \times 8/\underset{1}{\cancel{29}} = 16)$$

In working examples with students, be sure to have students explain how the key word(s) give direction concerning the operation needed to solve the problem.

ACTIVITY 2 - OUTCOMES 41, 42, 43, and 44

Point out that when word problems are being solved, it is not sufficient merely to identify the operation needed to solve the problem. Students must learn to identify pertinent information also. Provide students with a variety of problems. Suggest that students use a step-by-step procedure similar to the one that follows in solving word problems.

Example 1: A young man bought five hamburgers. Before arriving home, he had eaten three of the hamburgers. What fraction of the hamburgers were gone?

Solution:

1. What is known?
   5 hamburgers bought
   3 hamburgers eaten

2. What are the key words?
   What fraction ... gone

3. What is the mathematical question?
   3 is what part of 5

ACTIVITY 2 - OUTCOMES 41, 42, 43, and 44 (Cont'd)

4. What is the mathematical problem? Solve it.

   3 = □ x 5
   3 = 3/5 x 5

5. Is the answer reasonable?
   Answer = 3/5

   The answer is reasonable because a little more than one half of the hamburgers were eaten. 3/5 is a little more than 1/2.

Example 2: A football player gained 83 yards during a football game. During the same game he lost 12 1/2 yards. What was his total yardage for the game?

Solution:

1. What is known?
   83 yards gained
   12 1/2 yards lost.

2. What are the key words?
   total yardage for the game

3. What is the mathematical question?
   What is the sum of +83 and -12 1/2

4. What is the mathematical problem? Solve it.

$$\begin{array}{lrcr} \text{Add:} & +83 & = & +82\ 2/2 \\ & \underline{-12\ 1/2} & = & \underline{-12\ 1/2} \\ & & & 70\ 1/2 \end{array}$$

ACTIVITY 2 - OUTCOMES 41, 42, 43, and 44 (Cont'd)

5. Is the answer reasonable?
   Answer = 70 1/2

   The answer is reasonable because 12 1/2 is approximately 13. The total yardage would be approximately 83 - 13, or approximately 70.

Provide a variety of problems for student practice. Be sure to have students explain the steps taken. The answers to the 5 questions listed above may vary slightly from student to student. The variation in replies is acceptable, as long as all 5 questions are answered and all answers are accurate and mathematically correct.

AREA: FRACTIONAL NUMBERS: APPLICATIONS

OUTCOMES: 44. Solve a rate problem involving a fraction or a mixed number.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Multiply and divide whole numbers.
2. Multiply and divide fractions.
3. Set up and solve simple proportion problems.

Steps to Success

1. In order to succeed in solving these problems, students will need to recall some time equivalents such as 60 seconds = 1 minute, 52 weeks = 1 year.

2. Activity 1 directs students to solve all rate problems involving a fraction or a mixed number by setting up a proportion. A five-step, systematic approach is introduced. Activity 1 will be successful, however, only if students have received prior instruction on how to set up and solve proportion problems. See Mathematics Curriculum Guide: Grades 7-8, Area: Ratio, Proportions, and Percents - Outcome 10.

ACTIVITY 1 - OUTCOME 44

The problems listed below will provide practice for students. Have students follow the five-step process outlined. It is important for students to share their solutions. For any given problem, students may vary slightly in the ways they follow the five steps. Variations are acceptable as long as all steps are followed, a proportion is set up, and a correct answer is obtained.

Example 1: Class "Talker" - Mark can talk at a rate of 16 words every 5 seconds. How many words can he say in a minute and a half?

Solution:

1. What is known?
   16 words spoken every 5 seconds

2. What is the mathematical question?
   How many words spoken in a minute and a half

3. Ask the mathematical question using the language of proportions. Convert to equal units of measure if necessary.
   Conversion: 1 1/2 minutes = 90 seconds
   Proportion: 16 words is to 5 seconds as X words is to 90 seconds

4. Set up proportion and solve.

$$\frac{16}{5} = \frac{x}{90}$$

$$5x = 1440$$

$$x = 288$$

5. Is the answer reasonable?
   Yes. 16 words in 5 seconds is approximately 3 words per second.
   3 words per second for 90 seconds would be approximately 270 words. 288 is close to 270.

The word problems that follows offer additional practice for students. These problems can be personalized by substituting the names of actual students in class for the names used in the problems. Teachers and students may wish to create additional problems similar to these. Warn students that some problems may require that the answer be rounded (Example 5).

Example 1: The Class "Bookworm" - Doris can read 4 pages of a novel in 7 minutes. At this rate how long will it take her to read a chapter which is 26 pages long?

ACTIVITY 1 - OUTCOME 44 (Cont'd)

Example 2: Class "Paperthrower" - Paul hits the waste basket 4 times for every 6 wads of paper he throws. At the end of the week how many hits will he have if he tosses 30 wads of paper?

Example 3: "Good Math Student" - June gets the top score on a math test: 29 out of 30. If she always does about the same, how many points would she get on a 100 point math test?

Example 4: "Typist" - Amy can type 155 words in 5 minutes. How long would it take her to type a 2500 word English theme?

Example 5: "Eater" - Derek's favorite candy bars cost 25¢ for 3. How much does he pay for 20 of them?

AREA: INTEGERS: MEANING AND REPRESENTATION

OUTCOMES:

1. **Demonstrate an understanding of the definition by listing examples of the set of integers.**
2. **Given a number line, identify specific integers.**
3. **Given an integer state its opposite.**
4. **Define absolute value as the distance a number is from zero on the number line.**
5. **State the integer corresponding to a given absolute.**
6. **Simplify expressions that involve absolute value.**
7. **Identify the integer that is between two given integers.**
8. **Write inequalities using positive and negative integers.**

NOTES FOR THE TEACHER

Prerequisites for Students

1. Identify whole numbers.
2. Use a number line.
3. Read signed numbers.
4. Use the signs of inequality.

Steps to Success

1. The students´ ability to understand that there are as many numbers to the left of zero as there are to the right of zero is a key to interpreting integers on a number line.
2. Absolute value should be defined as the distance a number is from zero without regard to direction.
3. Ask students to note that as numbers move to the right on a number line, their values increase and as numbers move to the left on a number line, their values decrease.

ACTIVITY 1 - OUTCOMES 1, 2, and 3

Draw a number line on the board marking the line in equal increments. Locate zero, positive and negative numbers.

Associate the number line with examples from everyday life such as:

1. temperature on a thermometer;
2. profit and lost; or
3. loss and gain of yardage in football.

ACTIVITY 2 - OUTCOMES 4, 5, and 6

Define absolute value as the number of units a number is from zero on the number line.

On a number line, compare the distance of +4 and -4. Include examples to illustrate absolute value.

Show that $|4| = 4$ and $|-4| = 4$

$|5| = 5$ $\quad$ $|7 + 2| = 9$

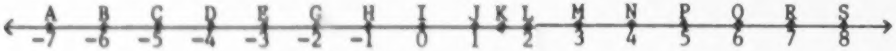
$|9 - 11| = 2$ $\quad$ $-|15 - 5| = -10$

$-2 + |-6| = 4$ $\quad$ $|10| - |+7| = 3$

Refer to the number line if needed.

ACTIVITY 3 - OUTCOMES 7 and 8

Draw a number line with points labeled with letters of the alphabet.



Ask questions about the number line such as:

1. What point corresponds to -2? (G)
2. What point is halfway between J and P? (M or 3)
3. Label each inequality as true or false:
   D < I (true)
   -3 < -1 (true)
   8 > -6 (true)

> In the examples above emphasize that:
>
> D < I means that D is to the left of I
> -3 < -1 means that -3 is to the left of -1
> 8 > -6 means that 8 is to the right of -6

4. Which symbols (=, <, >) are needed to make the following statements true?

   -5 _____ 0

   E _____ -3

   O _____ A

5. What is the approximate value of K? (1 1/2)

AREA: INTEGERS: COMPUTATION

OUTCOMES:
9. Add positive integers.
10. Add negative integers.
11. Add positive and negative integers.
12. Subtract positive integers.
13. Subtract negative integers.
14. Subtract positive and negative integers.
15. Multiply positive integers.
16. Multiply positive and negative integers.
17. Multiply negative integers.
18. Divide positive integers.
19. Divide a positive integer by a negative integer and vice versa.
20. Divide negative integers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Add, subtract, multiply, and divide whole numbers.
2. Have a knowledge of signed numbers.

Steps to Success

After completing the four activities described below, students should understand the following statements and apply these statements in computation problems involving integers.

1. When adding numbers with like signs, add the absolute values of the numbers and give the sum the common sign.

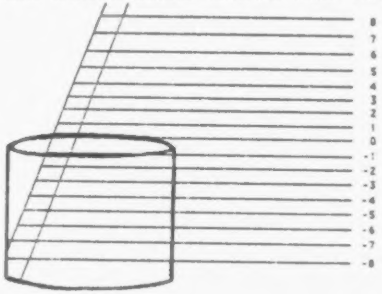
2. When adding numbers with unlike signs, subtract the absolute values of the numbers and give the difference the sign of the larger number.

NOTES FOR THE TEACHER (Cont'd)

3. When subtracting signed numbers, change the sign of the subtrahend and add.
4. When multiplying numbers with like signs, multiply the absolute values of the numbers and give the product a positive sign.
5. When multiplying numbers with unlike signs, multiply the absolute values of the numbers and give the product a negative sign.
6. When dividing numbers with like signs, divide the absolute values of the numbers and give the quotient a positive sign.
7. When dividing numbers with unlike signs, divide the absolute values of the numbers and give the quotient a negative sign.

The activities below describe situations which help students "see" why the statements above are true.

ACTIVITY 1 - OUTCOMES 9, 10, and 11



Draw a ladder in a hole.
Ask students to think of the step of the ladder located at ground level as zero.
All steps above ground level are positive.
All steps below ground level are negative.

Ask student to represent examples such as the following on the ladder:

(+1) + (+3) =   (+3) + (+7) =
(-7) + (-2) =   (-5) + (-3) =

Continue to give examples until students understand that adding numbers with like signs requires that the absolute values of the numbers be added and the sum be given the common sign.

Give examples such as the following and have students represent them on the ladder.

(+8) + (-2) =   (-7) + (+5) =
(+5) + (-6) =   (-8) + (+9) =

ACTIVITY 1 - OUTCOMES 9, 10, and 11 (Cont'd)

Continue to give examples until students understand that adding numbers with unlike signs requires that the absolute values of the numbers be added and the sum be given the sign of the larger number.

ACTIVITY 2 - OUTCOMES 12, 13, and 14

Ask students to work the following subtraction problem just as they were instructed in elementary school.

Subtract:

$$\begin{array}{r} 6 \\ \underline{2} \end{array}$$

Ask students what the answer is. (4)

Then, give students the problem below and instruct them to add the integers.

Add:

$$\begin{array}{r} 6 \\ \underline{-2} \end{array}$$

Ask students what the sum is. (+4)
Ask students to look at the two answers and to observe that they are the same. Then lead students to the understanding that subtracting a number is the same as adding its opposite. Provide examples for practice using every combination of positive and negative integers.

Example 1: Subtract: $(+8) - (+2)$

Solution: $(+8) - (+2) = (+8) + (-2) = +6$

Include examples such as the following in student practice.

$(+5) - (-2) =$ $\quad (-3) - (-6) =$

$(+6) - (+4) =$ $\quad (-8) - (+9) =$

ACTIVITY 3 - OUTCOMES 15, 16, and 17

Ask students to imagine that they are standing on a street corner as an emergency vehicle passes down the street. The word AMBULANCE is written across the side of the vehicle from left to right.

Reading is a positive motion (left to right)
Word is written in positive form (left to right)

positive (+) of X positive (+)

The result of a positive motion on a positive form is a positive image of AMBULANCE.

or

$$(+)(+) = +$$

Imagine that another ambulance comes down the street. This time you notice that the front has the letters ƎƆИA⅃UᙠMA written on it.

Reading is a positive motion (left to right)
Word is written backwards or in a negative form (right to left)

positive (+) of X negative (-)

The result of a positive motion on a negative form is a negative image of ƎƆИA⅃UᙠMA.

or

$$(+)(-) = -$$

Ask students to pretend further that they are in a car and that same ambulance comes up behind them. This time, the driver looks out of the rear view mirror and sees AMBULANCE written on the front of the vehicle. Why?

ACTIVITY 3 - OUTCOMES 15, 16, and 17 (Cont'd)

Reading through the rear view mirror (negative refection)		Word is written backwards or in a negative form (right to left)
negative (-)	of X	negative (-)

The result of a negative motion on a negative form is a positive image of AMBULANCE.

or

$(-)(-) = +$

Give examples of multiplication problems involving signed numbers.

Example: $(-8) \times (+4)$

Solution: $(-8) \times (+4) = -32$

When multiplying numbers with unlike signs, multiply the absolute values of the numbers and give the product a negative sign.

or

$(-)(+) = -$

Include examples similar to the following in student practice.

Multiply: $\begin{array}{r} +5 \\ \underline{+7} \end{array}$ Multiply: $\begin{array}{r} -20 \\ \underline{+\ 2} \end{array}$

Multiply: $(-3)(-18)$ Multiply: $(-2)(+1)(-3)=$

ACTIVITY 4 - OUTCOMES 18, 19, and 20

Explain to students that division is the inverse operation of multiplication. Since the product of numbers with like signs is positive, then the quotient of numbers with like signs is positive. Since the product of numbers with unlike signs is negative the quotient of numbers with unlike signs is negative.

Examples: If (+2) x (+3) = +6, then +6/+3 = +2 and +6/+2 = +3.
If (-2) x (-3) = +6, then +6/-3 = -2 and +6/-2 = -3.
If (-2) x (+3) = -6, then -6/+3 = -2 and -6/-2 = +3.

Give sufficient examples for students to demonstrate mastery of the rules for division of signed numbers.

AREA: INTEGERS: APPLICATIONS

OUTCOMES: 21. Translate word problems into expressions using integers.
22. Solve word problems involving integers.

AREA: WHOLE NUMBERS: APPLICATIONS

OUTCOMES: 22. Solve one-step and two-step problems.
23. Solve word problems.
25. Identify the number sentence appropriate to solve a two-step problem.
26. Identify the information needed to solve a problem.
27. Identify the information not needed to solve a problem.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know basic operations involving integers.

Step to Success

1. Use key words to determine the use of either a positive or negative number.

ACTIVITY 1 - OUTCOME 21

Ask students to tell which sign would be used to symbolize such words as loss, gain, profit, fall, etc.
Then give specific examples using these key words:
Loss of 5 yards (-5)
Gain of 10 yards (+10)
A $2,000 profit (+$2,000)
$100 "in the red" (-$100)
Ask students to add to the list of key words.

ACTIVITY 2 - INTEGERS: APPLICATIONS - OUTCOME 22
WHOLE NUMBERS: APPLICATIONS - OUTCOMES 22, 23, 25, 26, and 27

Provide students with word problems similar to the examples given.

Example 1: On each of three successive plays, the football team gained five yards, nine yards, and six yards. How many yards were gained in all on these three plays?

Solution: Discuss what sign is to be used and what operation should be used.

(+5) + (+9) + (+6) = +20

Example 2: In September, the school store made a profit of $200. In October and November, however, the store lost $100 and $75, respectively and in December, the store made a profit of $150. After the first three months of school, is the store operating at a profit or a loss? How much is that profit or loss?

Solution: (+200) + (-$100) + (-$75) = +$25.

The store shows a profit of $25.
The amount for December is unnecessary information and is not used to answer the problem.

Have students make up word problems similar to the ones above. Have students identify any unnecessary information contained in the problems.

AREA: INFORMAL GEOMETRY: TWO AND THREE DIMENSIONS

OUTCOMES:
1. **Identify and name points, lines, planes, segments, rays, intersecting lines, and parallel lines.**
7. **Identify center, chord, diameter, radius, arc, central angle, and inscribed angle of a circle.**

NOTES FOR THE TEACHER

The primary theme of the Informal Geometry unit is the introduction of geometric terms and their definitions. An attempt is made in Activity 1 to illustrate geometrical forms in everyday situations such as using a map of a town. A collection of visual objects to use as examples of definitions and concepts is helpful.

ACTIVITY 1 - OUTCOME 1

Place a map of the city on the bulletin board. On the map, place a pin at a shopping mall on one of the main roads through town. Explain that the pin indicates an exact location and that the pin is a symbol of a point. A geometric point is an exact location but it has no dimensions. Explain that a line is a set of points that extends in both directions. A symbol of a line might be a major highway that goes through town. It just keeps going in both directions. A line segment can be shown by placing two pins at different locations along the same road. Explain that segment means section, or a part; therefore, the distance between the two pins would represent a line segment (part of a line). The two pins represent the end points of the line segment. On the map locate a dead end street and place a pin there. That dead end street represents a ray. A ray is part of a line that begins at a point and goes from that point in one direction without end.

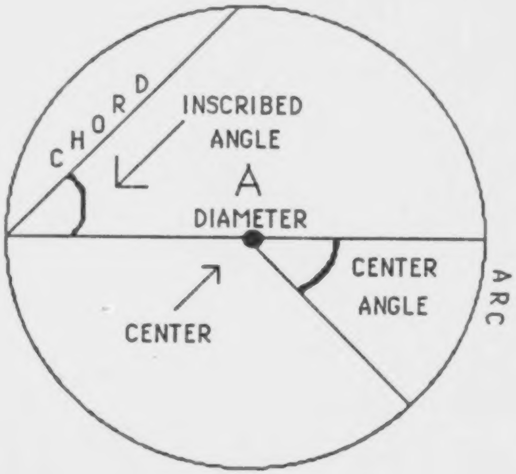
On the map find examples of two streets that never meet and are always equal distance apart. These streets can represent parallel lines. Intersecting lines can also be shown where two roads cross. Perpendicular lines can be shown where roads cross to form right angles. The map itself can represent a plane which is a flat surface that has no end. To illustrate a plane that has no end, other maps of adjacent sections of land, can be taped to the first map.

ACTIVITY 2 - OUTCOME 7

Explain that the compass can be used to draw a circle. Draw a circle on the board. Put a dot in the center. Indicate that the drawing is a circle. Point A is the center of the circle. All points on the circle are the same distance from point A. The distance around the circle is called the circumference. Any part of the circumference is called an arc. Illustrate an arc.

On the circle draw a line segment across the circle but not through the center. Explain that this line segment is a chord. Draw a diameter explaining that this is a special chord because it goes through the center of the circle. Draw a radius. Explain that a radius is a line segment that extends from the center of the circle to the edge. The radius is one-half the length of the diameter and the diameter is twice the length of the radius. An angle whose vertex is the center of the circle is a central angle. Explain that an inscribed angle of a circle is an angle whose vertex is on the circle.

Be sure that the drawing on the board is large enough for students to see how to spell the parts of a circle. Erase a word and let a student come to the board and rewrite it. Other students can attempt to spell the words correctly at their desks. Have students make a drawing of their own, correctly labeling the parts introduced in class.



AREA: INFORMAL GEOMETRY: TWO AND THREE DIMENSIONS

OUTCOMES:
2. Measure and classify angles as acute, right, or obtuse.
3. Identify congruent polygons and corresponding parts of the congruent polygons.
4. Classify triangles in terms of their sides and angles.
5. Classify polygons in terms of their sides and angles.
6. Recognize that the sum of the measures of the angles of a triangle is 180°.
8. Select the figure or state the name of the following plane geometric figures or lines: angle, right angle, triangle, square, rectangle, circle, lines, perpendicular lines, or parallel lines.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Identify a protractor as an instrument used to measure angles.
2. Identify the symbol for degree. (°).

Steps to Success

1. The student will need to practice drawing and reading with a protractor.

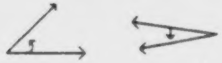
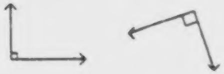
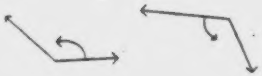
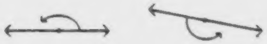
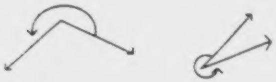
2. Since so many names must be memorized in the first three activities, mnemonic devices should be utilized to assist the memorization process.

ACTIVITY 1 - OUTCOME 2

Give each student a protractor. It is helpful to use an overhead projector when explaining the measurement of angles. Then, the teacher can use a protractor just like the students'. Have students observe the mark or arrowhead on the bottom, which will be placed on the vertex of an angle to be measured. Explain that either the inner scale or the outer scale can be used. The inner scale is used when one ray extends in the direction of the 0° mark on the inner scale. The outer scale is used when one ray extends in the direction of the 0° mark on

ACTIVITY 1 - OUTCOME 2 (Cont'd)

the outer scale. Demonstrate. Explain that there are five kinds of angles. Draw a table like the one below illustrating names, definitions, and examples.

ANGLE NAME	DEFINITION	EXAMPLE
Acute	contains less than 90°	
Right	contains 90°	
Obtuse	contains more than 90° but less than 180°	
Straight	contains 180°	
Reflex	contains more than 180°	

Students can remember the names of the angles by remembering the saying "A Rose Only Smells Rosy." The first letter of each word is the first letter of each angle in order from smallest to largest.

ACTIVITY 2 - OUTCOME 5

Explain that a polygon is a closed plane figure made up of line segments. The word polygon means many sides. The simplest polygon is a triangle because it takes at least three line segments to make a closed plane figure. The prefix in the name of the polygon tells how many sides the polygon has. The number of angles in a polygon is equal to the number of its sides.

Introduce these terms first:

Triangle- Tri means 3.	A tricycle has 3 wheels.
Quadrilaterals- Quad means 4.	Quadruplets are 4 children born in the same delivery.
Pentagon- Penta means 5.	The pentagon is a 5-sided building in Washington D.C.
Hexagon- Hex means 6.	

ACTIVITY 2 - OUTCOME 5 (Cont'd)

Next, introduce the following terms:

Septagon or Heptagon- Septa and Hepta mean 7.	Remember September.
Octagon- Octa means 8.	Remember October.
Nonagon- Nona means 9.	Remember November.
Decagon- Deca means 10.	Remember December.

Tell students that once the calendar contained only 10 months. At that time, September was the seventh month, October the eighth month, November the ninth month, and December the tenth month. Explain that some math books have "heptagon" and some say "septagon." Either word can be used for a seven sided polygon.

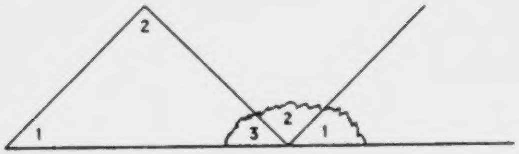
Students will be interested to know which two months were added to our calendar. Tell them that one was for Julius Caesar who was a Roman Emperor. Ask students which month do they think that was? (July) The other month was added for Caesar Augustus, who was really Augustus Caesar. Ask students which month do they think that was. (August)

ACTIVITY 3 - OUTCOME 4

Since the students have already studied angles they should have no trouble classifying triangles in terms of their angles. An acute triangle is one in which all the angles are acute or measure fewer than 90°. A right triangle contains only one right angle or a 90° angle. An obtuse triangle is a triangle that contains one angle that measures more than 90° but less than 180°. Show examples of each.

Explain that triangles can be classified also by the lengths of their sides. Associate names of triangles with examples such as the following: An equilateral triangle has all sides equal or congruent. Equilateral means equal sides. An isosceles triangle has at least two equal sides. To remember this name, tell students to think of a strong man like Hercules holding something heavy over his head. In the triangle formed by his arms and the object, the length of his arms would be the same. In that triangle, the two sides formed by his arms would be equal. (Hercules sounds like- Isosceles.) On the scalene triangle no sides are equal or congruent, so a different scale is needed for each. Draw examples of each triangle. In each drawing, label sides that are equal.

ACTIVITY 4 - OUTCOME 6



Cut out a triangle from a sheet of paper. Number each angle of the triangle. (1, 2, and 3) Cut off the three angles of the triangle and place them side by side as in the illustration. What kind of angle is formed? A straight angle is formed. Since the measure of a straight angle is 180°, the sum of the measures of the angles of a triangle is 180°.

ACTIVITY 5 - OUTCOME 3

Congruent polygons have the same size and shape. Cut out at least three triangles two of which are congruent. Place all the triangles on the board except one of the congruent triangles which can be held in your hand. Ask, students which of the triangles on the board has the exact same size and shape as the one held in your hand. Tell students to find out by placing the hand-held triangles on top of each of those on the board. See which pair is a perfect match. The two that match are congruent polygons. Explain also that the two sides that match are corresponding parts or sides.

ACTIVITY 6 - OUTCOME 8

On the board or on a worksheet, draw figures labeling them A, B, C, etc. Draw a line segment, a line, a ray, perpendicular lines, parallel lines, an angle, a right angle, a triangle, a square, a rectangle, a circle, and any other geometrical shape to be reviewed. As another exercise, make a list of the geometrical terms listed above and have students illustrate each.

GENERAL MATHEMATICS

AREA: INFORMAL GEOMETRY: TWO AND THREE DIMENSIONS

OUTCOME: 9. Identify prisms, pyramids, cylinders, cones, and spheres.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Be able to recognize circles, and parallelograms.

Steps to Success

1. A set of solid geometry figures is important to this activity. Bring a set to class or include in the activity that students build a set.

2. The ability to visually perceive a three dimensional drawing as a solid figure should be emphasized throughout the activity.

ACTIVITY 1 - OUTCOME 9

Bring to class a set of solid geometry figures. Pick up the rectangular prism. Ask what is it. Students may give several everyday answers. Explain that another name for it is a prism because its bases (top and bottom) are exactly the same (congruent- same size and shape) and parallel. The sides are all parallelograms. Then, show students a triangular prism explaining that the bases (top and bottom) are triangles but the sides are still parallelograms. Emphasize that prisms are named by the shape of their bases.

Next, consider the pyramids. A pyramid has only one base and this base may be a polygon of any number of sides; all the other faces are triangles that have a common vertex. A pyramid is also known by its base. A pyramid with a triangular base is called a triangular pyramid.

A cylinder is a solid figure (prism) with two congruent parallel bases that are circles. A can or oatmeal box is a good example of a cylinder. A cone is a solid figure with only one circular base. A sphere is a solid with all points an equal distance from the center. A ball would be an excellent example of a sphere.

ACTIVITY 1 - OUTCOME 9 (Cont'd)

Let students draw patterns to be cut out and glued to form solid figures. Have students compare the solid figures they have built.

GENERAL MATHEMATICS

AREA: INFORMAL GEOMETRY: TWO AND THREE DIMENSIONS

OUTCOMES:
10. Select the figure or state the name of the following solid geometric figures: sphere, cube, rectangular solid, cylinder, and cone.
11. Evaluate formulas.
12. Find the perimeter or circumference of a shape.
13. Find the perimeter of polygons up to six sides and units expressed in inches, feet, yards, centimeters, or meters.
14. Find the perimeter of rectangles, squares, and triangles expressed in word problems.
15. Find the perimeter of polygons when all dimensions of the figure are given.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Have a working knowledge of the four basic operation of whole numbers, decimals, and fractions.

Steps to Success

The key to success is being able to identify the correct formula and doing exactly what the formula indicates. The approach to finding perimeter, however, is to first see perimeter as the sum of the lengths of the sides. Then, an alternative formula is introduced as another way of finding the perimeter. For example, perimeter of a regular hexagon is first seen as s+s+s+s+s+s, and then as 6s.

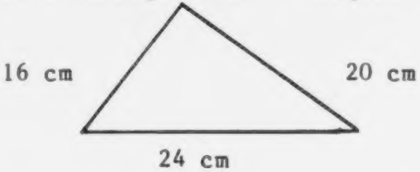
ACTIVITY 1 - OUTCOMES 10, 12, 13, 14, and 15

Formulas are like directions that tell how to put something together. The first thing to do is read the directions. Next, look at the pictures of all the parts. When reading a mathematical formula, look at all the variables, numerals, and signs of operation. What do the variables stand for? What operation is needed to complete the problem? When formulas (directions) are followed, and when the appropriate values for each variable are substituted, then the problem can be solved.

ACTIVITY 1 - OUTCOMES 10, 12, 13, 14, and 15 (Cont'd)

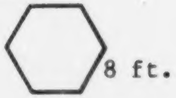
Continue with a discussion of perimeter. On the bulletin board, form a triangle using three tacks and yarn. The distance around it can be found by measuring the amount of string needed to go around the three tacks. Let students form other plane figures and show the distance around them. Ask students if there are other ways to find the distance around each figure. Explain that the distance around figure is called perimeter and that the perimeter can be found by adding the length of all the sides.

Example 1: Draw a triangle with 3 unequal sides.



Solution 1: Point out that the perimeter can be found by adding the 3 sides.
$p = s + s + s$
$p = 16\text{ cm} + 20\text{ cm} + 24\text{ cm}$
$p = 60\text{ cm}$

Example 2: Draw a regular hexagon.



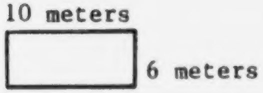
Solution 2: $p = s + s + s + s + s + s$
$p = 8\text{ ft.} + 8\text{ ft.} + 8\text{ ft.} + 8\text{ ft.} + 8\text{ ft.} + 8\text{ ft.}$
$p = 48\text{ ft.}$

As a "short cut" indicate that there are six equal sides of 8 ft. each. Or,

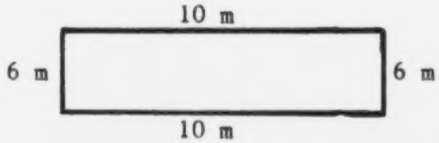
$p = 6 \times s$
$p = 6 \times 8$
$p = 48\text{ ft.}$

ACTIVITY 1 - OUTCOMES 10, 12, 13, 14, and 15 (Cont'd)

Example 3: How much fencing is needed to enclose a rectangular garden that measures 10 meters by 6 meters? Draw a picture of the fence.



Solution 3: Point out that students need to label all 4 sides, if they are to find the perimeter.



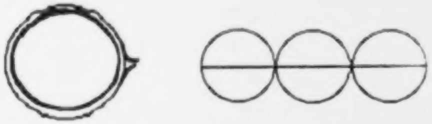
$p = s + s + s + s$
$p = 10\text{ m} + 6\text{ m} + 10\text{ m} + 6\text{ m}$
$p = 32\text{ m}$

As an alternative, students may see that half of the perimeter of the rectangle is 10 m + 6 m. Therefore, if that half of the perimeter is doubled, the total is generated.

$p = 2(l + w)$
$p = 2(10 + 6)$
$p = 32\text{ m}$

ACTIVITY 2 - OUTCOME 11

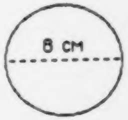
The distance around a circle is not called perimeter. It is called circumference. On the bulletin board place 3 lids. Cut a piece of string so that it just encircles one lid. Point out what happens when that string is stretched across the center of 3 adjacent lids. The illustration at the board shows that the circumference of one lid is slightly greater than the diameters of 3 lids.



Ask students if they think the same thing would happen if lids of different diameters were used. (yes) Ask students what would happen if the distance around a circle were measured and then that distance were divided by its diameter? Lead students to see that the result would be a little more than 3. Inform students that the Greek letter π (pi) is to show the relationship between the circumference and the diameter. The decimal 3.14 or the fraction 22/7 is used as approximations for π . Thus the formula $C = \pi d$.

circumference = pi(3.14) x diameter

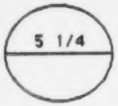
Example 1: Find the circumference of the circle.



Solution 1: $C = \pi d$
$C = 3.14 \times 8$
$C = 25.12$ cm

ACTIVITY 2 - OUTCOME 11 (Cont'd)

Example 2: Find the circumference of the circle.



Solution 2: $C = \pi d$

$C = 22/7 \times 5\ 1/4$

$C = \overset{11}{\cancel{22}}/\underset{1}{\cancel{7}} \times \overset{3}{\cancel{21}}/\underset{2}{\cancel{4}}$

$C = 33/2$

$C = 16\ 1/2$ ft.

AREA: LINEAR EQUATIONS AND INEQUALITIES: SOLVING EQUATIONS

OUTCOMES:
1. Use the standard order of operations to evaluate an expression involving more than one operation.
2. Evaluate an expression for a given value of the variable.
3. Evaluate a linear equation.
4. Solve an addition or subtraction equation by subtracting the same number from both sides or by adding the same number to both sides.
5. Solve a multiplication or division equation by dividing or multiplying both sides by the same number.
6. Solve two-step equations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Use the four basic operations on whole numbers, fractions, decimals, and integers.
2. Know the meaning of the words "evaluate", "solve", "variable", "equation", and "inequality."
3. Know the meaning of different forms of algebraic expressions.

ACTIVITY 1 - OUTCOME 1

Ask students if they have ever met "My Dear Aunt Sally."

Write this expression on the board, capitalizing the first letter in each word.

Explain that the first letter in each word represents one of the four basic operations (multiply, divide, add, subtract). This particular order of operations tells which operation should be used first in evaluating an expression such as:

$21 + 6 \div 2 - 7 \times 3$

ACTIVITY 1 - OUTCOME 1 (Cont´d)

My Dear Aunt Sally tells that students should first multiply or divide as indicated in the problem from left to right, and then, add or subtract as indicated in the problem from left to right.

Example: $21 + 6 \div 2 - (7 \times 3)$
$21 + (6 \div 2) - 21$
$(21 + 3) - 21$
$24 - 21$
3

ACTIVITY 2 - OUTCOMES 2 and 3

Remind students that most of them have worked with formulas. Ask them to remember examples such as $A = lw$ (for area of a rectangle) or $P = 2l + 2w$ or $P = 2(l + w)$ (for perimeter of a rectangle).

Ask students to suppose that they were asked to evaluate (find the value of) an expression.

Example: $5N$ if $N = 3$

Explain that just as they put values or numbers in the area and perimeter formulas, they do the same thing for algebraic expressions such as the one above.

Example: $A = P + I$
A = Amount of money, P = principal, and I = Interest.

Let P = \$500 and I = \$25. Show how \$500 is put in the place of P and \$25, in the place of I. Add the two values together as indicated by the plus sign to find the value of A.

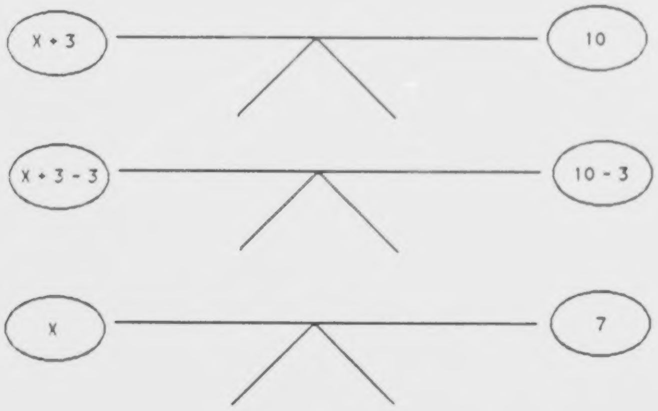
Solution: $A = \$500 + \$25$
$A = \$525$

ACTIVITY 3 - OUTCOMES 4 and 5

From the science lab secure a triple beam balance (scales with 2 balance pans). When the scales are exactly balanced and the needle points to the center, correlate the two balance pans with the two members of the equation. Use coins of the same denomination on each side to get the pans balanced. Show that when a coin is added to one side, the same size coin must be added to the other side to keep the pans balanced. Now subtract coins of like denomination from each side to show that the scales will still be balanced. Perform similar actions to illustrate that whatever is done to one side must be done to the other in order to keep the scales balanced.

The same thing is true for equations. Whether adding, subtracting, multiplying, or dividing, the same operation must be performed on each side of an equation to keep the equation "balanced."

Example:



Solution: Subtract 3 from both sides and pans are still balanced. Work examples involving subtraction, multiplication, and division.

ACTIVITY 4 - OUTCOME 6

One way to treat the solution of equations such as $2X + 3 = 11$ is to say that the variable X has a contagious disease and has to be "isolated." To do this, students need to perform the needed operation (add, subtract, multiply, or divide) to isolate the variable.

Example: $2X + 3 = 11$

Solution: $2X + 3 - 3 = 11 - 3$

$$2X = 8$$

Now, proceed to "isolate" X by multiplying or dividing, as necessary.

$$2X = 8$$

$$\frac{2X}{2} = \frac{8}{2}$$

$$X = 4$$

Point out that X has been "isolated." Explain further that its value (4) has been found and the equation has been solved.

AREA: LINEAR EQUATIONS AND INEQUALITIES: PROBLEM SOLVING

OUTCOMES:
7. **Write an expression involving a variable for a given mathematical phrase or situation.**
8. **Solve problems by writing equations involving addition, subtraction, multiplication, or division and solving the equation.**

NOTES FOR THE TEACHER

Prerequisite for Students

1. Competence in evaluating expressions and solving equations.

Steps to Success

Students should be able to translate words into mathematical expressions or equations by recognizing relevant facts and key words. Relevant facts and key words serve as clues to the correct operational symbol(s) to be used in the expression or equation.

ACTIVITY 1 - OUTCOME 7

Write several word expressions on the board using different key words or phrases such as "more than," "less than," "the product of," "times as much," etc. Then write the algebraic expression below it, asking for the students' help in translating to symbols.

Example 1: Seven more than a number

7 and N

$7 + N$

ACTIVITY 1 - OUTCOME 7 (Cont´d)

Example 2: The product of seven and a number.

(multiply) 7 N

$7 \times N$

Example 3: Seven less than a number.

N - 7

When subtracting numbers, the larger number is written first. If 7 is less than a number, then the number(N) must be larger than 7 and must be written first.

Other examples could include phrases such as "increased by," "decreased by," "the quotient of," "the sum of," "the difference between."

ACTIVITY 2 - OUTCOME 8

Continue the exercise done in ACTIVITY 1 for algebraic expressions to include terms such as "equals," "is equal to," "is the same as," "is," etc.

Example: Five more than twice a number is nineteen.

$5 + 2N = 19$

After equations are written, have students solve them.

AREA: LINEAR EQUATIONS AND INEQUALITIES: SOLVING INEQUALITIES

OUTCOMES:
9. Graph inequalities on the number line.
10. Solve an addition or subtraction inequality involving rational numbers.
11. Solve a multiplication or a division inequality involving rational numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have knowledge of the number line.
2. Have experience in solving equations.
3. Understand "greater than" or "less than" symbols.

Step to Success

1. Students should be able to graph inequalities, solve the solution set and then draw the graph of the solution set.

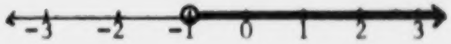
ACTIVITY 1 - OUTCOME 9

Solution sets include more than whole numbers. They may include real numbers also. When the real numbers are solutions, fractions and decimals are included. When showing this on a graph, it is not possible to place a dot on the number line to represent every possible solution.

To graph a solution set on a number line, show the starting point with a solid dot to include the point or an open dot to exclude the point. Then draw an arrow to the left or right to show which direction the solution set includes.

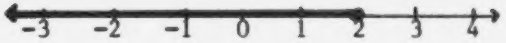
Let the students describe the solution sets that are represented by graphs such as:

Example 1:

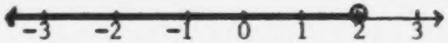


$(X > -1)$

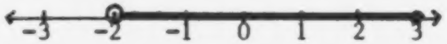
ACTIVITY 1 - OUTCOME 9 (Cont'd)

Example 2: 

($x \leq 2$)

Example 3: 

($x < 2$)

Example 4: 

($-2 < x \leq 3$)

Let the students graph inequalities such as:

Example 1: $x \geq 4$

Example 2: $y < 1$

Example 3: $w \leq -2$

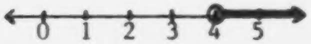
ACTIVITY 2 - OUTCOMES 9, 10, and 11

The only difference in solving inequalities and linear equations is when both sides of the inequality are divided or multiplied by a negative number. Multiplying or dividing both sides of the inequality by a negative number will reverse the order of the inequality. (See Example 4.)

Example 1: $a + 2 > 6$

$a + 2 - 2 > 6 - 2$

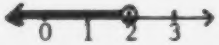
$a > 4$



Example 2: $b - 5 < -3$

$b - 5 + 5 < -3 + 5$

$b < 2$

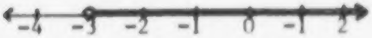


ACTIVITY 2 - OUTCOMES 9, 10, and 11 (Cont'd)

Example 3: $2C > -6$

$$\frac{2C}{2} > \frac{-6}{2}$$

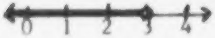
$$C > -3$$



Example 4: $-2c > -6$

$$\frac{-2C}{-2} > \frac{-6}{-2}$$

$$C < 3$$



GENERAL MATHEMATICS

AREA: MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

OUTCOMES:
1. Recognize equivalent customary linear units.
2. Recognize equivalent metric linear units.
10. Recognize equivalent customary mass units.
11. Recognize equivalent metric mass units.
12. Choose the appropriate customary measure of mass and find equal measures for given measures of mass.
13. Choose the appropriate metric measure of mass and find equal measures for given measures of mass.
16. Recognize equivalent customary capacity units.
17. Recognize equivalent metric capacity units.
18. Choose the appropriate customary measure of capacity and find equal measures for given measures of capacity.
19. Choose the appropriate metric measure of capacity and find equal measures for given measures of capacity.
29. Choose the appropriate metric measure for linear, mass, or capacity measure, for a given situation.
30. Choose the appropriate customary measure for linear, mass, or capacity measure, for a given situation.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know the customary units.

Step to Success

1. Emphasize that the metric system is based on tens and is easier to learn that the customary system because it is systematic.

ACTIVITY 1 - OUTCOMES 1, 2, 10, 11, 12, 13, 16, 17, 18, 19, 29, and 30

Explain the following to the students.

Many years ago man had the need for a standard of weight. He took one grain from the very center of the wheat stem. He called this a grain. It took exactly 5760 of these grains to make what we call a pound. Through the years man has used many methods of measuring. At this point, ask the class for examples of different methods of measure such as cloth material measured from the tip of the nose to the finger tips for the yard.

Explain further that as years passed, man changed standards of measure to suit his own business or trade. But for everyone's best interest, a system had to be set so that everyone would get equal measures. One of the simplest systems man developed is called the metric system.

The metric system has three basic units of measure.

1. liter - to measure volume
2. gram - to measure weight
3. meter - to measure length

Compare this to the customary units we use.

1. liter - to measure volume
    - pint
    - quart
    - gallon
2. gram - to measure weight
    - ounce
    - pound
    - ton
3. meter - to measure length
    - inch
    - foot
    - yard
    - mile

Use containers and other measuring devices to illustrate the use of these units of measure.

ACTIVITY 1 - OUTCOMES 1, 2, 10, 11, 12, 13, 16, 17, 18, 19, 29, and 30 (Cont'd)

Make a set of flash cards that ask what unit of measure would be more reasonable to use.

Example 1:

(METRIC UNITS)
A Match Box of Sand

grams

back of card

Example 2:

(CUSTOMARY UNITS)
Length of the Classroom

yards and/or feet

back of card

AREA: MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

OUTCOMES:
3. Measure line segments or objects in a figure in customary and metric units.
4. Measure line segments or objects in a figure in customary and metric units requiring the addition of two lengths.
24. Identify the measuring scale for temperature in metric measure as degrees Celsius.
25. Identify the measuring scale for temperature in customary measure as degrees Fahrenheit.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the basic fractional concepts.
2. Know the symbol used to mean degrees of temperature (°).

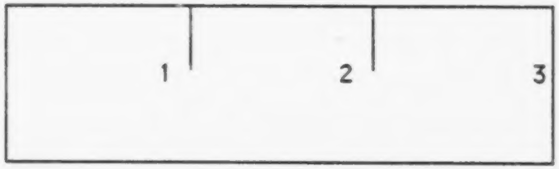
Steps to Success

1. The ability to transfer the knowledge of fractional parts to a ruler will enhance success.

2. Fahrenheit and Celceius scales have fixed points for boiling and freezing. The scale between the points are different.

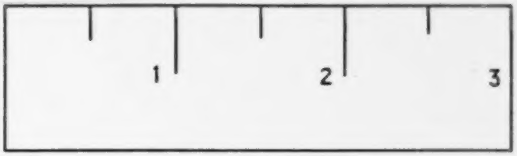
ACTIVITY 1 - OUTCOMES 3 and 4

Model 1:

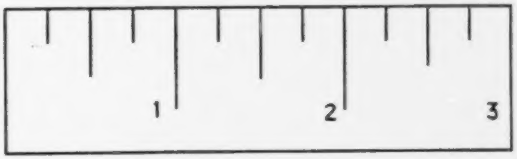


ACTIVITY 1 - OUTCOMES 3 and 4 (Cont'd)

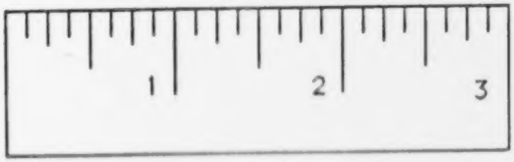
Model 2:



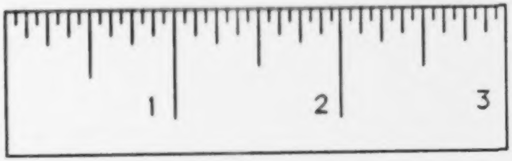
Model 3:



Model 4:



Model 5:



ACTIVITY 1 - OUTCOMES 3 and 4 (Cont'd)

Display the Model 1 Ruler.

Discuss objects whose lengths could be measured with this ruler.

Display the Model 2 Ruler.

Discuss the division mark (1/2) and how it helps to measure objects more accurately. Name the length that each mark represents on the ruler.

Display the Model 3 Ruler.

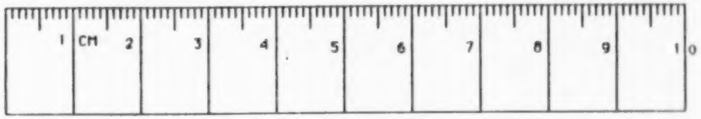
Discuss its division marks (1/4) and how these marks make it more convenient to measure other objects. Name the lengths that each mark represents on the ruler.

Repeat the same process with the Model 4 and Model 5 rulers.

Display these five rulers one under the other. Emphasize the equivalent fractional parts of the rulers by placing a yard stick vertically over them such as 1/2 = 2/4 = 4/8 = 8/16.

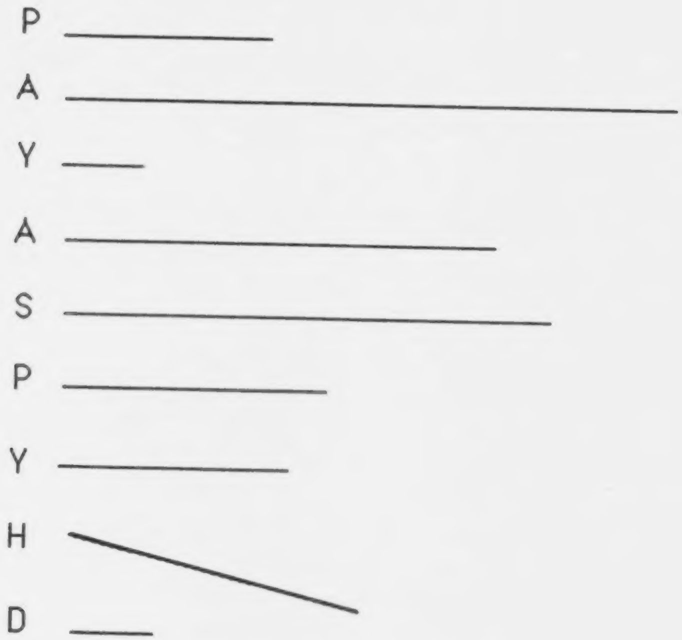
ACTIVITY 2 - OUTCOMES 3 and 4

Construct a metric ruler and discuss the divisions on it.



ACTIVITY 2 - OUTCOMES 3 and 4 (Cont'd)

Make a game using lettered lengths to be measured. Letters should spell out any appropriate phrase for students.

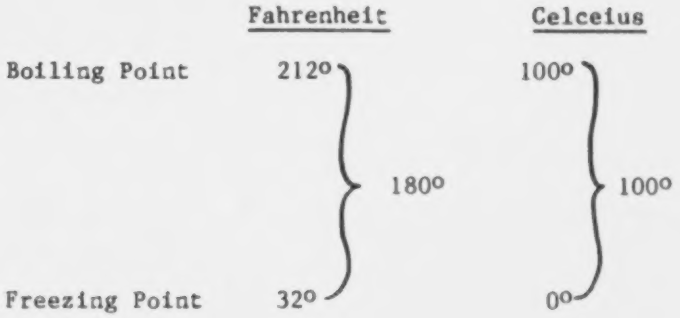


____ 1 3/4 in. ____ 6 1/2 cm ____ 1 1/4 in. ____ 4 cm ____ 13 mm ____ 1/2 in. ____ 3 5/8 in. ____ 3 1/2 cm ____ 2 7/8 in.

The student measures each line and fills in the blanks to spell out "HAPPY DAYS."

ACTIVITY 3 - OUTCOMES 24 and 25

Draw the following diagram.


Fahrenheit
Celceius
Boiling Point
212°
100°
180°
100°
Freezing Point
32°
0°

Lead a discussion to develop the concept of the difference in the two scales using the numbers 100 and 180.

The Celceius is 180/100 or 9/5 of the Fahrenheit scale. The Fahrenheit is 100/180 or 5/9 of the Celceius scale.

Let students construct one large thermometer with a Fahrenheit scale on one side and a Celceius scale on the other side. Display it in the room where the temperature of the day can be compared on both scales over a period of several weeks.

GENERAL MATHEMATICS

AREA: MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

OUTCOMES:
5. Find the area of a rectangle, square, parallelogram, triangle, trapezoid, and a circle.
6. Find the surface area of a prism, a pyramid, a cylinder, a cone, and a sphere.
7. Solve word problems relating to rectangular area involving customary or metric units of inches, feet, yards, centimeters, or meters.
8. Solve problems requiring the calculation of rectangular area when figures are used and lengths of all sides are labeled.
9. Find the volume of a prism, a pyramid, a cylinder, a cone, and a sphere when the dimensions are given in customary or metric units.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Perform the four basic operations with whole numbers, fractions, and decimals.
2. Identify plane figures as well as prisms.

Steps to Success

1. The ability to interpret and use formulas will be helpful.

2. Emphasize that area is always some version of length times width and is always expressed in square units.

3. Volume is some version of length times width times height. Volume is always found in cubic units.

ACTIVITY 1 - OUTCOMES 5, 7, and 8

Have students read the following note:

Go home this afternoon after school.
Eat the cake I have left for you.

ACTIVITY 1 - OUTCOMES 5, 7, and 8 (Cont´d)

Mow the neighbor´s yard.
Collect $15.00 if you finish.

Ask students to analyze the message.

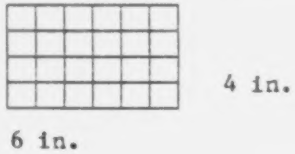
It tells someone what to do step-by-step. What are the consequences for leaving out a step?

A formula is a message. It tells exactly what to do with the values of variables in order to achieve a task.

Consider the following messages as the 6 area formulas are discussed.

Message: To find the AREA OF A RECTANGLE, multiply length times width.

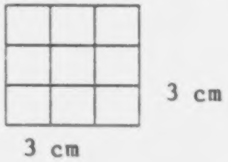
1. RECTANGLE



$A = l \times w$
$A = 6 \times 4$
$A = 24$ sq. in.

Message: To find the AREA OF A SQUARE, multiply side times side.

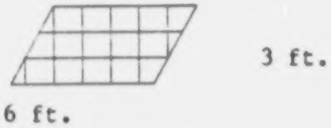
2. SQUARE



$A = x^2$
$A = 3^2$
$A = 3 \times 3$
$A = 9$ sq. cm

Message: To find the AREA OF A PARALLELOGRAM, multiply base times height.

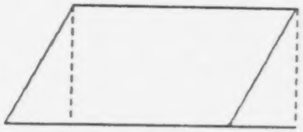
3. PARALLELOGRAM



$A = b \times h$
$A = 6 \times 3$
$A = 18$ sq. ft.

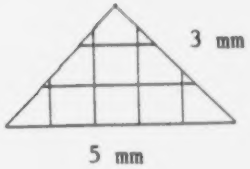
ACTIVITY 1 - OUTCOMES 5, 7, and 8 (Cont'd)

Show how a parallelogram has the same area as a rectangle.



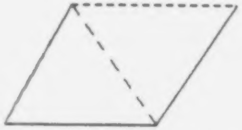
Message: To find the AREA OF A TRIANGLE take 1/2 of the product of base and height.

4. TRIANGLE



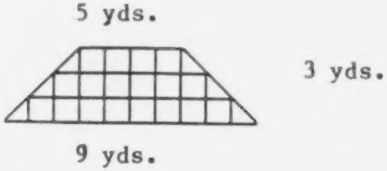
A = 1/2 x b x h
A = 1/2 x 5 x 3
A = 15/2
A = 7 1/2 sq. mm

Show a triangle is 1/2 a parallelogram.



Message: To find the AREA OF A TRAPEZOID take 1/2 the sum of the bases and multiply by the height.

5. TRAPEZOID

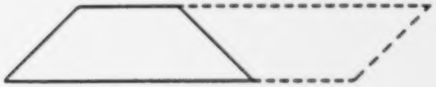


$$A = \frac{B + b}{2} \times h$$

A = 1/2 x (B + b) x h
A = 1/2 x (9 + 5) x 3
A = 1/2 x (14) x 3
A = 7 x 3
A = 21 sq. yds.

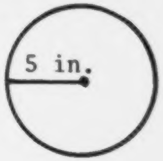
ACTIVITY 1 - OUTCOMES 5, 7, and 8 (Cont'd)

Show how a trapezoid is 1/2 a parallelogram.



Message: To find the AREA OF A CIRCLE, multiply π times the square of the radius.

6. CIRCLE



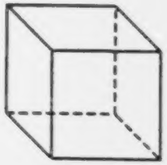
$A = \pi \times R^2$
$A = 3.14 \times 5^2$
$A = 3.14 \times 25$
$A = 78.5$ sq. in.

ACTIVITY 2 - OUTCOME 6

Explain that the surface area is the part of a prism, pyramid, cylinder, cone, or sphere that would be covered if the surfaces of a solid figure were to be printed.

Let the students help develop the message and the formula for the surface area of a cube, a rectangular solid, and a pyramid.

1. Message: The surface area of a cube is the area of one side multiplied by 6. (6 sides)



$A = 6\ s^2$

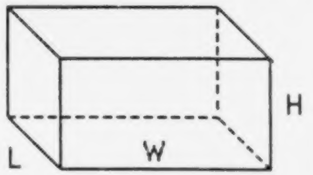
Example: Find the surface area of a cube whose side is 3 decimeters.

$A = 6\ s^2$
$A = 6 \times 3^2$
$A = 6 \times 9$
$A = 54$ sq. dm

ACTIVITY 2 - OUTCOME 6 (Cont'd)

2. Message: The surface area of a rectangular solid is the area of the top and bottom plus the area of the two sides plus the area of the two ends.

Rectangular Solid



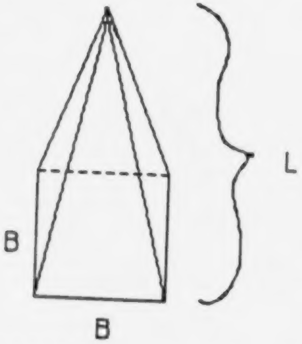
$$A = \underset{\text{Bottom and top}}{2lw} = \underset{\text{Two sides}}{2lh} + \underset{\text{Both ends}}{2wh}$$

Example: Find the surface area of a rectangular solid whose length is 6 ft., width is 4 ft., and height is 2 ft.

Solution:
$A = 2lw + 2lh + 2wh$
$A = 2(6)(4) + 2(6)(2) + 2(4)(2)$
$A = 48 + 24 + 16$
$A = 88$ sq. ft.

3. Message: The surface area of a pyramid is the area of its base plus the area of its four sides.

Pyramid



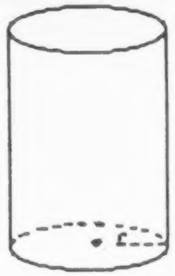
$A = b^2 + 4(1/2\ bh)$

Example: Find the surface area of a pyramid whose base is 3 m and height is 4 m.

Solution:
$A = b^2 + 4(1/2bh)$
$A = 3^2 + 4(1/2 \times 3 \times 4)$
$A = 9 + 4(6)$
$A = 33$ sq. m

ACTIVITY 2 - OUTCOME 6 (Cont'd)

4. Message: The surface area of a cylinder is the area of the 2 circles that serve as its top and bottom ($2\pi r^2$) plus the area of the circular wall that would be a rectangle if it were cut and laid flat. The rectangle has a length of πd or $2\pi r$ (circumference of a circle) and a height of h.



Example: Find the surface area of a cylinder whose height is 6 yds. and radius of the base is 2 yds.

Solution: The formula for the surface area of a cylinder is:

$$A = 2\pi r^2 \times 2\pi r(h)$$

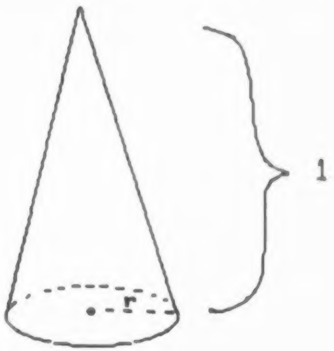
Rewritten by factoring out the $2\pi r$, the formula for the surface area of the cyinder becomes:

$$A = 2\pi r(h + r)$$

Apply the formula.

$$A = 2\pi r(h + r)$$
$$A = 2(3.14)(2)(6 + 2)$$
$$A = 2 \times 3.14 \times 2 \times 8$$
$$A = 100.48 \text{ sq. yds.}$$

5. Message: The surface area of a cone is found by using the formula $A = \pi r^2 + \pi rl$



Example: Find the surface area of a cone whose slant height (1) is 5 mm and the radius of the base is 1.5 mm.

Solution:

$$A = \pi r^2 + \pi rl$$
$$A = (3.14)(1.5)^2 + 3.14(1.5)(5)$$
$$A = 3.14 \times 2.25 + 3.14 \times 7.5$$
$$A = 7.065 + 23.55$$
$$A = 30.615 \text{ sq. mm}$$

ACTIVITY 2 - OUTCOME 6 (Cont'd)

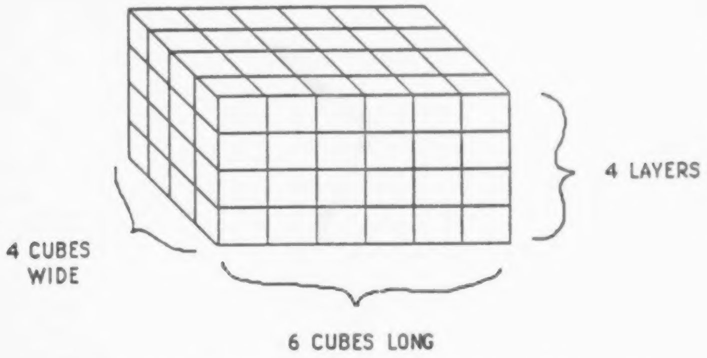
6. Message: The surface area of a sphere is found by using the formula $A = 4\pi r^2$

Example: Find the surface area of a sphere whose radius is 12 meters.

Solution: $A = 4\pi r^2$
$A = 4 \times 3.14 \times 12^2$
$A = 4 \times 3.14 \times 144$
$A = 1808.64$ sq. m

ACTIVITY 3 - OUTCOME 9

The formula for finding volume is derived by counting the number of cubes in a solid figure. An example is a rectangular solid that is 4 x 6 x 4.



4 x 6 = 24 cubes in one layer.
24 x 4 = 96
96 cubes in 4 layers.

Therefore, $V = l \times w \times h$

NOTE: All volume is measured in cubic units.

257

ACTIVITY 3 - OUTCOME 9 (Cont'd)

The following are formulas for finding the volume of these geometric solids.

Cube $V = s^3$
or $V = s \times s \times s$

Cylinder $V = \pi r^2 h$

Sphere $V = 4/3\, \pi r^3$

Cone $V = 1/3\, \pi r^2 h$

Pyramid $V = 1/3\, Bh$, where B = area of the base

Give examples such as:

1. What is the volume of an engine cylinder whose radius is 2 in. and whose height is 5 in.?

$$V = \pi r^2 h$$
$$V = 3.14 \times 2^2 \times 5$$
$$V = 3.14 \times 4 \times 5$$
$$V = 62.80 \text{ cu. in.}$$

2. How many cubic feet of space are inside a square, pyramid-shaped tent whose base is 10 ft. and whose height is 15 ft.?

$$B = s^2$$
$$B = 10^2$$
$$B = 100$$

$$V = 1/3\, Bh$$
$$V = 1/3(100)15$$
$$V = 500 \text{ cu. ft.}$$

AREA: MEASRUEMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

OUTCOMES:
14. Add and subtract with metric measures. (mass)
15. Add and subtract with customary measures. (mass)
20. Add and subtract with metric measures. (capacity)
21. Add and subtract with customary measures. (capacity)
22. Solve word problems involving capacities with units of ounces, pints, quarts, gallons, milliliters, or liters.
23. Solve word problems involving capacities but without conversions between metric and customary units and without using formulas to determine volume.
26. Add and subtract with metric measures. (temperature)
27. Add and subtract with customary measures. (temperature)
28. Solve word problem involving computation of metric measures or customary measures.
31. Solve problems by giving sensible answers.
32. Solve word problems using the appropriate customary or metric measures.
33. Add and subtract measures of time and compute elapsed time.
34. Solve problems using addition, subtraction, multiplication, and division for units of time to include years, months, weeks, days, hours, and minutes.
35. Solve problems using fractional units of time involving denominators of 2, 3, 4, 5, 6, 10, and 12.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Perform the four basic operations with whole numbers, fractions, and decimals.

Steps to Success

1. All like units of measure should be kept in the same column when adding or subtracting.

2. Units of measure should be simplified if possible.

ACTIVITY 1 - OUTCOMES 14, 15, 20, 21, 26, 27, 33, and 34

Ask students if they remember how their elementary school teacher taught them to add. Demonstrate with an example similar to the following:

$$\begin{array}{r} 29 \\ +48 \\ \hline \end{array}$$

First add the units $\begin{array}{r} 9 \\ +\ 8 \\ \hline 17 \end{array}$

$$\begin{array}{r} 1 \\ 29 \\ +48 \\ \hline 7 \end{array}$$

17 is more than 1 ten, so write down 7 and carry the 1 ten. (Regroup)

$$\begin{array}{r} 1 \\ 29 \\ +48 \\ \hline 77 \end{array}$$

Now add the 1 ten to the other numbers in the tens column.

77 is the sum.

Adding units of measure involves a similar procedure.

Example 1:

$$\begin{array}{r} 2\text{ ft. }9\text{ in.} \\ +4\text{ ft. }8\text{ in.} \\ \hline \end{array}$$

First add the inches $\begin{array}{r} 9\text{ in.} \\ +\ 8\text{ in.} \\ \hline 17\text{ in.} \end{array}$

$$\begin{array}{r} 1\text{ ft.}\phantom{\text{ 9 in.}} \\ 2\text{ ft. }9\text{ in.} \\ +4\text{ ft. }8\text{ in.} \\ \hline 5\text{ in.} \end{array}$$

17 in. is more than 1 foot
(1 ft = 12 in.), so write down the 5 inches remaining and carry the 1 foot.

$$\begin{array}{r} 1\text{ ft.}\phantom{\text{ 9 in.}} \\ 2\text{ ft. }9\text{ in.} \\ +4\text{ ft. }8\text{ in.} \\ \hline 7\text{ ft. }5\text{ in.} \end{array}$$

Now, add the feet.

Therefore the answer is 7 ft. 5 in.

ACTIVITY 1 - OUTCOMES 14, 15, 20, 21, 26, 27, 33, and 34
(Cont'd)

Ask students to consider elementary subtraction problems.

$$\begin{array}{r} 84 \\ \underline{-57} \end{array}$$

Subtract the units first.

$$\begin{array}{r} 7\,14 \\ \cancel{8}4 \\ \underline{-57} \end{array}$$

Borrow 1 ten and add the ten to the 4 in the units place.

$$\begin{array}{r} 7\,14 \\ \cancel{8}4 \\ \underline{-57} \\ 27 \end{array}$$

Now subtract.

A similar procedure is used to subtract units of measure.

Example 1:

$$\begin{array}{rr} 8\text{ ft.} & 4\text{ in.} \\ \underline{-5\text{ ft.}} & \underline{7\text{ in.}} \end{array}$$

Subtract the inches first.

$$\begin{array}{rr} 7\text{ ft.} & 16\text{ in.} \\ \cancel{8}\text{ ft.} & \cancel{4}\text{ in.} \\ \underline{-5\text{ ft.}} & \underline{7\text{ in.}} \end{array}$$

Borrow 1 foot.
Since 1 foot equals 12 inches we add 12 in. to 4 in. and get 16 in.

$$\begin{array}{rr} 7\text{ ft.} & 16\text{ in.} \\ \cancel{8}\text{ ft.} & \cancel{4}\text{ in.} \\ \underline{-5\text{ ft.}} & \underline{7\text{ in.}} \\ 2\text{ ft.} & 9\text{ in.} \end{array}$$

Now subtract.

Consider the algorithm for multiplication next.

$$\begin{array}{r} 28 \\ \underline{\times\ 7} \end{array}$$

Multiply seven times the 8 in the ones place and get 56.

ACTIVITY 1 - OUTCOMES 14, 15, 20, 21, 26, 27, 33, and 34
(Cont'd)

$$\begin{array}{r} 5 \\ 28 \\ \times\ 7 \\ \hline 6 \end{array}$$

Put down the 6 and carry the 5.

$$\begin{array}{r} 28 \\ \times\ \ 7 \\ \hline 196 \end{array}$$

Multiply 7 x 2 in the tens place and get 14. Then add the 5 that was carried to get 19.

Now a similar procedure for multiplying units of measure.

Example 2:

$$\begin{array}{rl} 2 \text{ lb. } 8 \text{ oz.} \\ \times \qquad\quad 7 \\ \hline \end{array}$$

Multiply 7 times 8 oz. and get 56 oz.
Since 56 oz. is 3 lb. and 8 oz.,

$$\begin{array}{r} 3 \text{ lb. } 8 \text{ oz.} \\ +14 \text{ lb.} \qquad \\ \hline 17 \text{ lb. } 8 \text{ oz.} \end{array}$$

Put down the 8 oz. and carry the 3 lb.
Multiply 7 times 2 lb. and get 14 lb.

Therefore, 2 lb. 8 oz. x 7 = 17 lb. 8 oz.

Consider the algorithm for division next.

$$3\overline{)726}$$

$$\begin{array}{r} 2 \\ 3\overline{)726} \\ \underline{6} \end{array}$$

Divide 3 into 7, and multiply the trial quotient times divisor.

$$\begin{array}{r} 24 \\ 3\overline{)726} \\ \underline{6} \\ 12 \\ \underline{12} \end{array}$$

Divide 3 into partial dividend 12, and multiply partial quotient times divisor 3.

ACTIVITY 1 - OUTCOMES 14, 15, 20, 21, 26, 27, 33, and 34
(Cont'd)

$$\begin{array}{r} 242 \\ 3\overline{)726} \\ \underline{6} \\ 12 \\ \underline{12} \\ 06 \\ \underline{6} \end{array}$$

Subtract and bring down next number. Divide partial quotient by divisor and multiply partial dividend times divisor, then subtract.

Use a similar procedure for dividing units of measure.

$$3\overline{)7\text{ yd. }2\text{ ft. }6\text{ in.}}$$

$$\begin{array}{l} 2 \\ 3\overline{)7\text{ yd. }2\text{ ft. }6\text{ in.}} \\ \underline{6\phantom{\text{ yd. 2 ft. 6 in.}}} \\ 1\text{ yd.} \end{array}$$

Divide divisor (3) into partial dividend (7). Multiply partial quotient (2) times divisor (3) and get (6). Subtract and get remainder 1 yd.

$$\begin{array}{l} 2\text{ yd.} \\ 3\overline{)7\text{ yd. }\;2\text{ ft. }6\text{ in.}} \\ \underline{6\text{ yd.}\phantom{\text{ 2 ft. 6 in.}}} \\ \underline{1\text{ yd.}=3\text{ ft.}} \\ \phantom{3)1\text{ yd.}=}5\text{ ft.} \end{array}$$

Convert 1 yd. to 3 feet and add to partial dividend of 2 feet.

$$\begin{array}{l} 2\text{ yd. }\;1\text{ ft.} \\ 3\overline{)7\text{ yd. }\;2\text{ ft. }6\text{ in.}} \\ \underline{6\text{ yd.}\phantom{\text{ 2 ft. 6 in.}}} \\ \underline{1\text{ yd.}=3\text{ ft.}} \\ \phantom{3)1\text{ yd.}=}5\text{ ft.} \\ \phantom{3)1\text{ yd.}}\underline{-3\text{ ft.}} \\ \phantom{3)1\text{ yd.}=}2\text{ ft.} \end{array}$$

Divide divisor of 3 into partial dividend of 5 feet. Get 1 ft. Multiply trial quotient times divisor of 3 and get 3 ft. Subtract and get 2 ft.

ACTIVITY 1 - OUTCOMES 14, 15, 20, 21, 26, 27, 33, and 34
(Cont'd)

```
      2 yd. 1 ft.  10 in.
   3/7 yd. 2 ft.    6 in.
     6 yd.
     1 yd. 3 ft.
           5 ft.
          -3 ft.
           2 ft.= 24 in.
                  30 in.
                  30 in.
                   0
```

Since the remainder 2 ft. = 24 in. add the 24 in. to the 6 in. and get 30 in.
Divide the divisor of 3 into the partial quotient of 30 and get 10.
Multiply partial quotient of 10 times divisor of 3 and get 30 and subtract.

Give students practice with all types of examples for adding, subtracting, multiplying, and dividing customary and metric units.

ACTIVITY 2 - OUTCOMES 22, 23, 28, 31, 32, and 35

Review adding, subtracting, multiplying, and dividing of units of measure, if necessary. Review converting units of measure, if necessary. Give example problems such as:

1. What unit of measure would be used with apples?
2. What unit of measure would be used to measure a bowl of water?
3. What unit of measure would be used to measure your temperature?
4. Would it be more sensible to measure ice cream in ounces or meters?
5. Would it be more sensible to measure candy in liters or grams?

Give students other word problems such as the following:

Example 1: Mr. Brown needs eight servings of steak. He has a 9 lb. 8 oz. piece of beef. For each person to have an equal share, what size pieces should he cut?

ACTIVITY 2 - OUTCOMES 28, 31, 32, and 35 (Cont´d)

1 lb. 3 oz.
8/9 lb. 8 oz.
8
1 = 16
24
24

Therefore, Mr. Brown should cut the steaks 1 lb. 3 oz .each.

Example 2: A coach wanted to spend 2/3 of practice time on one drill. How much time can be spent on one drill if practice lasts for 1 hr. 45 min.?

Method A: Since 1 hr. 45 min. = 1 45/60 = 1 3/4

2/3 x 1 3/4 =

2/3 x 7/4 = 14/12 = 1 2/12 = 1 1/6

The answer is 1 1/6 hr., or 1 hour 10 minutes, since 10 minutes is 1/6 of 60 minutes.

Method B: Since 1 hr. 45 min. = 105 min.

2/3 x 105/1 = 210/3 = 70 min.

Therefore, one drill will be 70 minutes or 1 hr. 10 min.

Example 3: One play dough recipe calls for 1 cup flour, 1 1/2 cups water, 1/2 teaspoon cream of tartar, and 2 cups salt. If the recipe is tripled, how much of each ingredient will be needed?

Solution:
1 c flour x 3 = 3 c flour
1 1/2 c water x 3 = 4 1/2 c water
1/2 t cream of tartar x 3 = 1 1/2 t cream of tartar
2 c salt x 3 = 6 c salt

GENERAL MATHEMATICS

AREA: MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS

OUTCOMES:

36. **Change units of linear, capacity, and weight measures by converting within the customary system.**
37. **Change units of linear, capacity, and weight measures by converting within the metric system.**
38. **Convert linear customary units of measure of inches, feet, and yards or metric units of measure of millimeters, centimeters, meters, and kilometers.**
39. **Convert measures of customary capacity units of quarts and gallons or metric capacity units of milliliters and liters.**
40. **Convert measures of customary weight units of ounces and pounds or metric weight units of grams and kilograms.**

NOTES FOR THE TEACHER

Prerequisites for Students

1. Multiply and divide by 10, 100, and 1,000 by moving the decimal.
2. Multiply fractions.

Steps to Success

1. Students may have difficulty in converting metric units simply because they do not know the prefixes. Encourage all students to learn the following 6 prefixes in order:

   kilo, hecto, deka, base, deci, centi, and milli

2. Other conversions may be made by using conversion factors.

ACTIVITY 1 - OUTCOMES 37, 38, 39, and 40

Emphasize that there are three bases in the metric system: liter, gram, and meter.

ACTIVITY 1 - OUTCOMES 37, 38, 39, and 40

Ask students what multiplying any number by 10 does to the number. Multiplying by 100. Multiplying by 1,000.

Relate these questions to moving the decimal point.

Then, ask students questions like the following:

What does dividing by 10 do to a number?

What does dividing by 100 do?

What does dividing by 1,000 do?

Relate these questions to moving the decimal.

Explain that no matter what base or unit of measure is being worked with in the metric system, simply multiply or divide by 10, 100, or 1,000 to convert units.

Draw the following diagram:

kilo (km)	hecto (hm)	deka (dam)	Base	deci (dm)	centi (cm)	milli (mm)
kilogram	hectogram	dekagram	gram (g)	decigram	centigram	milligram
kilometer	hectometer	dekameter	meter (m)	decimeter	centimeter	millimeter
kiloliter	hectoliter	dekaliter	liter (l)	deciliter	centiliter	milliliter
1000 grams	100 grams	10 grams	1 gram	.1 gram	.01 gram	.001 gram
1000 meters	100 meters	10 meters	1 meter	.1 meter	.01 meter	.001 meter
1000 liters	100 liters	10 liters	1 liter	.1 liter	.01 liter	.001 liter

Go over the chart thoroughly so that students understand the progressions to the left and right of the base.

The students may use the sentence, Kids Have Dirty Boots During Circus Months to remember the order of the prefixes. Explain that when changing metric units, the decimal is moved to the left or right as many places as it takes to get the new unit.

ACTIVITY 1 - OUTCOMES 37, 38, 39, and 40 (Cont'd)

Example 1: Change 38 milligrams to grams.

Solution 1: Look at the prefixes:

kilo, hecto, deka, gram, deci, centi, milli

Start with the milli prefix and count to the left 3 places to get to grams. This indicates that the decimal in the given number 38 must be moved three places to the left.

.038.

Therefore, 38 milligrams is equal to .038 grams.

Example 2: Change 2 hectoliters to liters.

Solution 2: Look at the prefixes.

kilo, hecto, deka, liters, deci, centi, milli

Start at the prefix hecto and count to the right 2 places to get to the base liter.

Therefore, we will move the decimal two places to the right in 2 hectoliters.

2.00.

Two hectoliters is equal to 200 liters.

Give students other problems to convert such as:

1. 6 cm to mm
2. 27 dm to cm
3. 285 m to cm
4. 1.9 km to dam
5. 400 cm to m

ACTIVITY 2 - OUTCOMES 36, 37, 38, 39, and 40

Make up a system of measure. Use units such as A, B, C, D, and E. The conversion chart would look like this:

1A = 2B
1B = 4C
1C = 6D
1D = 3E

It is necessary to follow these rules to be able to convert units of measurement.

1. Put the unit to be changed in a fraction form with 1 as its denominator.
2. Place a multiplication sign to the right of the fraction.
3. Construct the conversion factor by placing the unit changed from in the denominator and the unit changed to in the numerator.
4. Multiply the fractions.

Example 1: Change 6A to B´s.

Solution 1: Step 1.

$$\frac{6A}{1} \quad \begin{array}{l}\text{unit to be changed}\\ \text{denominator}\end{array}$$

Step 2.

$$\frac{6A}{1} \times$$

Step 3.

$$\frac{6A}{1} \times \frac{2B}{1A} \quad \begin{array}{l}\text{unit changed to}\\ \text{unit changed from}\end{array}$$

Step 4.

$$\frac{6A}{1} \times \frac{2B}{1A} = \frac{12\,\cancel{A}B}{1\cancel{A}} = \frac{12B}{1} = 12B$$

Therefore, 6A = 12B.

ACTIVITY 2 - OUTCOMES 36, 37, 38, 39, and 40 (Cont´d)

Example 2: Change 8C to D´s.

Solution 2: $\frac{8C}{1} \times \frac{6D}{1C} = \frac{48\cancel{C}D}{1\cancel{C}} = 48D$

Therefore, 8C = 48D

Example 3: Change 21C to B´s.

Solution 3: $\frac{21C}{1} \times \frac{1B}{4C} = \frac{21B\cancel{C}}{4\cancel{C}} = 5\frac{1}{4}B$

Therefore, 21C = $5\frac{1}{4}$ B

Explain that if no conversion factor is available to change to directly, a two or more step process will be needed.

Example 4: Change 7A to C´s.

Solution 4: $\frac{7A}{1} \times \frac{2B}{1A} \times \frac{4C}{1B} = 56C$

Consider the following units of measure.

1 oz. = 8 dr.
1 pt. = 16 oz.
1 qt. = 2 pt.
1 gal. = 4 qt.

Example 1: Convert 8 gal. to quarts.

Solution 1: $\frac{8\text{ gal.}}{1} \times \frac{4\text{ qt.}}{1\text{ gal.}} = \frac{32\text{ qt.}}{1} = 32$ qt.

Therefore, 4 gal. = 32 qt.

ACTIVITY 2 - OUTCOMES 36, 37, 38, 39, and 40 (Cont'd)

Example 3: Convert 96 oz. to quarts.

Solution 3: $\frac{96 \text{ oz.}}{1} \times \frac{1 \text{ pt.}}{16 \text{ oz.}} \times \frac{1 \text{ qt.}}{2 \text{ pt.}} = \frac{96 \text{ qt.}}{32} = 3 \text{ qt.}$

Therefore, 96 oz. = 3 qt.

AREA: PROBABILITY AND STATISTICS

OUTCOMES:

1. Make a prediction for an outcome.
2. Determine probability using experiments.
3. Use a graph or a table to organize data and determine the range for a set of data.
4. Identify the mean, the median, and the mode for a set of data.
5. Given a frequency table, determine the number of cases on which a score distribution is based.
6. Given a frequency table, name the number of scores at or below a given score.
9. Find the probability of single and multiple outcomes.
10. Determine the probability of independent and dependent events.
11. Make a prediction for an outcome.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the meaning of "outcome" and "probability."
2. Understand ratios.
3. Arrange numbers in order.
4. Find the average of a group.

Steps to Success

1. The key to understanding the probability of an event is the students ability to recognize the possible outcomes and to carry out simple experiments to determine probability.

2. Range, mean, median, and mode must be defined.

3. Before using a graph or a table to organize data, the data must first be ordered.

ACTIVITY 1 - OUTCOMES 1, 2, and 11

Ask students if they have heard of people who predict the future. These people are sometimes called psychics. Explain that there are other people who need to predict events. For example, the owner of a store predicts (or estimates) how many of a certain item he can sell when he orders his goods. How does he make his prediction? Point out that the store owner bases his prediction on past events. Explain that the probability of future events can be predicted by experimenting. The possible outcome must be known before the probability of an event occurring can be predicted.

For example, if a coin is flipped, how many different outcomes are possible?

There are only two possible outcomes. Either the coin will land showing "heads" or "tails."

Divide the class into four groups. Each group will have each student flip a coin 3 times. One member of each group will record the number of "heads" and "tails" shown. Totals will be added at the board.

The experiment shows that two results were recorded: "heads" and "tails." Introduce students to the probability formula:

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Explain that if the desired outcome is heads, the probability is 1/2 because there is 1 favorable outcome (heads) and 2 possible outcomes (heads and tails).

Repeat this type of experiment using a die. Ask if anyone can predict the probability of rolling a three on a single die? The probability is 1/6 since there is one favorable outcomes and six possible outcomes. Ask what the probability is of tossing a seven? Since there is no seven on a die, the number of favorable outcomes is zero. Therefore, the probability of tossing a seven is 0 (0/6 = 0).

ACTIVITY 2 - OUTCOME 9

ACTIVITY 1 can be expanded to include problems with single and multiple outcomes. Review the formula used in ACTIVITY 1.

$$\text{Probability (P)} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Point out that in single outcome problems, only one favorable outcome is desired.

Example 1: What is the probability of tossing 5 on a die?

Solution 1: $P = 1/6$

Explain that in multiple outcome problems, more than one favorable outcome is named.

Example 2: What is the probability of tossing either a 2 or 3 on a die?

Solution 2: $P(3 \text{ or } 2) = P(3) + P(2)$

$P(3 \text{ or } 2) = 1/6 + 1/6$

$P(3 \text{ or } 2) = 2/6$ or $1/3$

Work with students to make up additional multiple-outcome problems.

ACTIVITY 3 - OUTCOME 10

The probability of independent events occurring can be calculated using simple probability formulas. Start by asking how students could determine the probability of flipping a coin with "heads" and rolling a die to show 3. Direct students to look at all of the possible outcomes of flipping a coin and rolling a die. Let "H" = heads and "T" = tails. Let the numerals 1-6 stand for what can be shown when a die is rolled. List the possible combinations for the coin flip and the die toss.

289

288

Example:

H1	T1
H2	T2
H3	T3
H4	T4
H5	T5
H6	T6

There are 12 possible outcomes but only 1 favorable outcome. Therefore, the probability of tossing a heads and rolling a 3 is 1/12.

This solution helps to show that the formula for the probability of independent events occurring is valid. The formula is P(A and B) = P(A) x P(B).

Determine the probability of dependent events varies slightly. For dependent events, the first event has some effect on the probability of the second event.

Example: Suppose a bag contains 10 marbles: 7 red, 2 white, and 1 black. What is the probability of picking a red marble on the first draw and a white marble on the second draw? (The first draw is not put back in the bag.)

Solution: The probability of picking a red marble first is equal to $\frac{\text{favorable outcomes}}{\text{total possible outcomes}}$ or $\frac{7}{10}$.

Do not replace the marble that was picked. Ask students if the probability of drawing a white marble on the next pick is equal to 2/10? Explain that the answer is no, because there are only 9 marbles left in the bag. Therefore, the probability of choosing a white marble on the second pick is [illegible]9.

The probability of picking a red marble on the first draw and a white marble on the second draw is

$$\frac{7}{10} \times \frac{2}{9} = \frac{14}{90} \quad \text{or} \quad \frac{7}{45}.$$

ACTIVITY 4 - OUTCOMES 3, 4, 5, and 6

Ask students to count the number of coins in their pocket or purse. Record the various numbers of coins on the board using a table such as the following:

No coins	//	2	3 coins	////	5	Over 5 coins	//// //	7
1 coin	/	1	4 coins	///	3			
2 coins	///	3	5 coins	////	4			

Ask students to:

A. find the range 7 - 1 = 6

B. find the mean $\frac{2 + 1 + 3 + 5 + 3 + 4 + 7}{7} = \frac{25}{7} = 3.57$

C. find the mode over 5 coins

D. find the median 4 coins

Explain that the table used to organize the data is called a frequency table. Using the table, give students problems like the following:

1. Find the number of cases by adding the frequency of each score.

   2 + 1 + 3 + 5 + 3 + 4 + 7 = 25

2. Find the number of people who have fewer than 4 coins.

   2 + 1 + 3 + 5 = 11

292 293

AREA: PROBABILITY AND STATISTICS

OUTCOMES: 7. Solve word problems requiring the interpretation of a picto-, bar, line, or circle graph and the use of information contained in the graph.
8. Read and construct tables or graphs used to organize data.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Interpret four kinds of graphs (picto-, bar, line, and circle graphs).
2. Perform computations involving percents, fractions, and ratios.
3. Follow specific steps when solving word problems.
4. Know the number of degrees in a circle (360°).
5. Find a given percent of 360°.
6. Use a protractor to construct an angle.

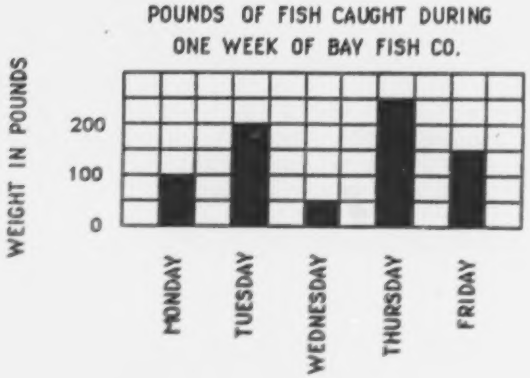
Steps to Success

1. The key to solving word problems involving graphs is to interpret correctly the graphs and the information contained in the graphs.

2. The key to constructing circle graphs is to understand the given data and transforming the given data into a percent of a circle.

ACTIVITY 1 - OUTCOME 7

Construct a bar graph such as the one below.



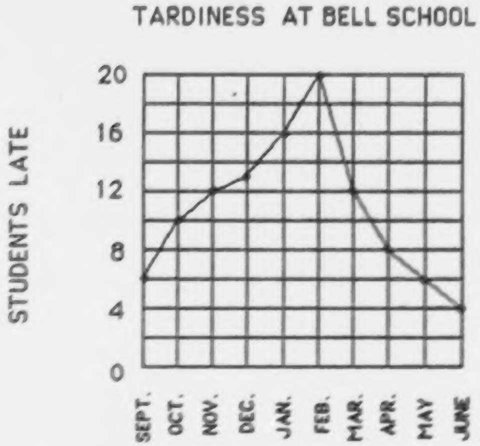
Direct students to use the graph to solve problems similar to the following:

Example: Find the average daily catch of the Bay Fishing Co. during the week shown on the graph.

Solution:	Monday	100 lbs.	150	150 lbs. is the average daily catch.
	Tuesday	200 lbs.	5/750	
	Wednesday	50 lbs.	5	
	Thursday	250 lbs.	25	
	Friday	150 lbs.	25	
		750 lbs.	0	
			0	

ACTIVITY 2 - OUTCOME 7

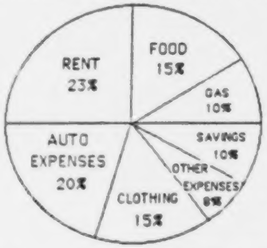
Construct a line graph similar to the one below. Tell students to use the graph to answer questions regarding tardiness at Bell School.



1. How many students were late in December? (13)
2. During which month was the most tardiness? (Feb.) least? (June)
3. On the average, how many students were tardy each month during the school year?
   $6 + 10 + 12 + 13 + 16 + 20 + 12 + 8 + 6 + 4 = \frac{107}{10} = 10.7$

ACTIVITY 3 - OUTCOME 7

Construct a circle graph similar to the one below. Tell students to use a graph such as the one below to answer questions.



Example 1: If John's income is $1,500.00 per month, how much does he spend for rent?

Solution 1:

$$\begin{array}{r} 1500.00 \\ \underline{\times \quad .23} \\ \$\ 345.00 \end{array}$$

Have students write other questions that can be answered from information contained in the graph.

ACTIVITY 4 - OUTCOME 8

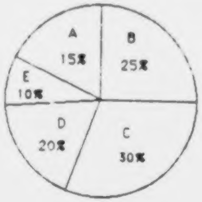
Ask students to construct a circle graph to illustrate the following math test scores:

15% - A 25% - B 30% - C 20% - D 10% - F

Show students how to find a given percent of a 360° circle: 10% of $360^\circ = 36^\circ$ 15% of $360^\circ = 54^\circ$

20% of $360^\circ = 72^\circ$ 25% of $360^\circ = 90^\circ$ 30% of $360^\circ = 108^\circ$

Show students how the translation of percents into degrees makes it possible to draw graphs that accurately represent the information given.



ACTIVITY 4 - OUTCOME 8 (Cont'd)

Ask students to construct another circle graph to illustrate the situation described below.

Example: Bob's income is $1,000.00 per month. His monthly expenses are as follows:

Rent	$250.00
Food	$200.00
Auto Expenses	$200.00
Utilities	$100.00
Savings	$ 50.00
Clothing	$ 50.00
Miscellaneous	$150.00

Draw a circle graph representing Bob's monthly budget.

Solution: If needed, show students how to express percents as degrees of a circle.

Rent = 25% or 90°

Food = 20% or 72°

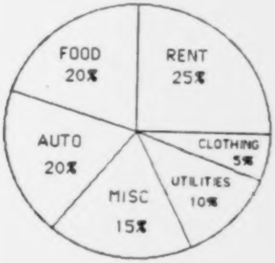
Auto = 20% or 72°

Savings = 5% or 18°

Utilities = 10% or 36°

Clothing = 5% or 18°

Misc. = 15% or 54°



AREA: RATIO AND PROPORTION

OUTCOMES:
1. Solve word problems requiring the calculation of the ratio of two numbers.
2. Solve a proportion in which the unknown is a numerator in one ratio and a denominator in the other ratio.
3. Solve a proportion in which a denominator is unknown.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Perform the four basic operations on whole numbers, decimals, and fractions.
2. Write fractions in their simplest form.
3. Follow directions.
4. Understand equivalent proportions.

Steps to Success

The key to solving word problems using ratios is being able to write the ratio in the order named and being able to recognize different ways to write a ratio.

ACTIVITY 1 - OUTCOME 1

Use school situations to solve word problems in calculating the ratio of two numbers. For example, ask for the number of girls in the class and then for the number of boys. Show these numbers as a ratio of girls to boys. Then, ask for the ratio of boys to girls. If there are 13 girls and 15 boys the ratio of girls to boys is 13 to 15 or 13:15 or 13/15 or 13 ÷ 15. The ratio of boys to girls is 15:13 or 15 to 13 or 15/13 or 15 ÷ 13.

Use other examples such as red heads to blondes, red shirts to blue shirts, and students to teachers.

ACTIVITY 2 - OUTCOME 1

Devise word problems such as the following:

Example: The school basketball team played 10 games. They won 6 games. What is the ratio of games won to the games played? (6 to 10) What is this ratio in simplest form? (3 to 5) What is the ratio of number of games lost to the number of games won? (4 to 6) or (2 to 3 in simplest form)

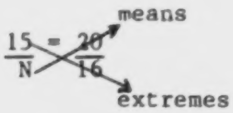
ACTIVITY 3 - OUTCOMES 2 and 3

Explain that equivalent ratios are called proportions, and that they are the same as equivalent fractions. The product test can be used to prove equivalent proportions, just as it is used to prove equivalent fractions.

Explain further that all proportion problems are not complete. Ask students to consider an example such as:

$$\frac{15}{N} = \frac{20}{16}$$

Ask students if they can find a number to replace N that will make the proportion true or equivalent. Explain that the product of the extremes must equal the product of the means. Define means and extremes if needed.



$$\frac{15}{N} = \frac{20}{16}$$

$$\begin{aligned} 15 \times 16 &= 240 \text{ (product of extremes)} \\ N \times 20 &= 20N \text{ (product of means)} \\ 20N &= 240 \\ N &= 12 \end{aligned}$$

Show that it does not matter which number is missing.

$$\frac{6}{10} = \frac{N}{25}$$

$$\begin{aligned} 6 \times 25 &= 150 \text{ (product of extremes)} \\ N \times 10 &= 10N \text{ (product of means)} \\ 10N &= 150 \\ N &= 15 \end{aligned}$$

AREA: RATIO AND PROPORTION

OUTCOMES:
4. **Solve word problems relating to scale drawings using whole numbers.**
5. **Use scale drawings and proportions to determine actual lengths.**

NOTES FOR THE TEACHER

Prerequisites for Students

1. Use the four basic operations on whole numbers, decimals, and fractions.
2. Set up a proportion and solve it.

Step to Success

1. The key to success is understanding which numbers belong to the scale and which numbers go with the actual object.

ACTIVITY 1 - OUTCOMES 4 and 5

Explain the definition: If the ratio of each length or distance in a drawing to the corresponding length or distance on the actual object is the same, then the drawing is called a scale drawing. The ratio used is called its scale.

On the chalkboard draw a rectangle measuring 15 inches by 10 inches. This will represent a scale drawing of a classroom. Tell students that the scale is "1 inch = 2 feet" and ask them to find the length and and width of the classroom. Explain that the first number (inch) refers to the drawing and the second number (2 feet) refers to the actual classroom.

Ask students to look for other examples in their textbooks. For example, in their science book they may find a drawing of a cell. The scale then might be 250 mm = 1 mm. The drawing would be bigger than the actual cell so the first number in the scale (250 mm) refers to the drawing and the second number (1 mm) refers to the cell.

ACTIVITY 2 - OUTCOMES 4 and 5

Bring in blueprints or maps. Have students determine different distances. Have students build a scale model of their school or some famous building they are studying in social studies.

AREA: RATIO AND PROPORTION

OUTCOMES:
6. Write decimals as percents and percents as decimals.
7. Demonstrate an understanding that a percent is actually the ratio of a number to 100.

AREA: DECIMAL: CONVERSION TO FRACTIONS AND PERCENT

OUTCOMES:
38. Rename a decimal fraction or a mixed decimal, up to three digits, as a percent or vice versa.
39. Select a fraction or a mixed number equivalent to a given decimal.
40. Select a decimal equivalent to a given fraction or mixed number.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Understand decimals and the place value system.

Step to Success

1. The key to success is being able to take something you know and relate it to a new situation.

ACTIVITY 1 - AREA: RATIO AND PROPORTION - OUTCOMES 6 and 7
AREA: DECIMAL: CONVERSION TO FRACTIONS AND PERCENT - OUTCOMES 38, 39, and 40

Explain: Percent really means per hundred. Ask how many cents are in a dollar. Ask how many years are in a century. Explain that instead of writing the word percent the symbol "%" can be used.

$0.25 and 25c are two ways to write 25 cents. 25¢ means 25 out of 100 because there are 100 cents per dollar. 25 out of 100 can be written as a ratio: 25 to 100, 25:100, or 25/100.

ACTIVITY 1 - AREA: RATIO AND PROPORTION - OUTCOMES 6 and 7
AREA: DECIMAL: CONVERSION TO FRACTIONS AND PERCENT - OUTCOMES 38, 39, and 40
(Cont´d)

Since percent means per hundred, 25/100 = 25%; therefore, 25% = 0.25 = 25/100.

Explain that there are at least two ways to remember how to change a percent to a decimal or visa versa. (1) Make up a simple example like the one above. Observe what happens to the decimal point if the % sign is removed. It moves two places to the left. Then all problems in which a percent is to be changed to a decimal are done the same way. There is a simple way to determine which way to move the decimal when changing a decimal to a percent. (2) Another way to remember is using the saying "A baby makes a puddle in a diaper." Abbreviate puddle as PDL. which means to change a PERCENT to a DECIMAL and move the decimal to the LEFT. Abbreviate diaper as DPR. which means to change a DECIMAL to a PERCENT and move the decimal to the RIGHT. Move the decimal two places in each case because percent means per hundred and hundredths has two places.

Examples: Percents to Decimal - Puddle PDL

29% = 0.29
182% = 1.82
0.06% = 0.0006

Decimals to Percent - Diaper DPR

0.32 = 32%
3.6 = 360%
0.086 = 8.6%
3 = 300%

GENERAL MATHEMATICS

AREA CONSUMER MATHEMATICS

OUTCOMES:

1. Solve word problems requiring the calculation of: the final cost when the discount, or sales tax rate,and item costs are given; the amount of discount, tax, commission, or downpayment when the rate and total cost are given; or the ratio of two whole numbers.
2. Solve word problems requiring: calculation of the total purchase price and determination of the correct change; determination of change when the purchase price and the amount tendered are given; recognition of equivalent amounts using different combinations of coins and currency.
3. Solve word problems requiring: the selection of the best of four purchases of the same item; determination of savings when the same item is offered at different prices.
4. Solve word problems requiring: computation of the interest charged on a credit purchase; computation of the price difference between a credit purchase and a cash purchase; comparing cost of cash and credit purchase from the information given in an advertisement.
5. Solve word problems requiring: solution of realistic problems involving time and money; use of information given in chart form.
6. Given a blank check or money order, determine the correct placement of a given item of information.
7. Given a completed check or money order, determine incorrect placement or error in placement of information.
8. Solve word problems requiring: computation of net pay when salary and deductions are given; computation of net pay when amount of deductions, the rate of pay, and the number of hours worked are given.
9. Solve word problems for: miles per gallon when given distance in miles and gallons of fuel used; time when given distance traveled and average rate of speed; rate when given distance traveled and time.

NOTES FOR THE TEACHERS

These outcomes are discussed thoroughly in the mathematics section of the Alabama High School Graduation Exam Basic Skills Instructional Materials (BSIM). Any further development of these outcomes would be a natural extension of the materials contained in that document.

OUTCOME 1 - BSIM Competency I-7
OUTCOME 2 - BSIM Competency VI-1
OUTCOME 3 - BSIM Competency VI-2
OUTCOME 4 - BSIM Competency VI-3
OUTCOME 5 - BSIM Competency VI-4
OUTCOMES 6 and 7 - BSIM Competency VI-5
OUTCOME 8 - BSIM Competency VI-7
OUTCOME 9 - BSIM Competency VI-6

AREA: WHOLE NUMBERS: APPLICATIONS

OUTCOMES:

24. Solve rate problems.
28. Solve a problem requiring the calculation of an average.
29. Solve word problems to determine miles per gallon when given distance in miles and gallons of fuel used.
30. Solve word problems to determine time when given distance traveled and average rate of speed.
31. Solve word problems to determine rate when given distance traveled and time.
32. Solve word problems involving computation of net pay when salary and deductions are given in only dollar amounts.
33. Solve word problems involving computation of net pay when amounts of deductions, the rate of pay, and the number of hours worked are given.
34. Solve word problems requiring the interpretation of a picto-, bar, line, or circle graphs and the use of information contained in the graphs.

NOTES FOR THE TEACHERS

These outcomes are discussed thoroughly in the mathematics section of the Alabama High School Graduation Exam Basic Skills Instructional Materials (BSIM). Any further development of these outcomes would be a natural extension of the materials contained in that document.

OUTCOMES 24, 29, 30, and 31 - BSIM Competency VI-6
OUTCOME 28 - BSIM Competency I-9
OUTCOMES 32 and 33 - BSIM Competency VI-7
OUTCOME 34 - BSIM Competency III-1

AREA: DECIMALS: APPLICATIONS

OUTCOMES:
47. Solve word problems involving realistic applications requiring addition, subtraction, multplication, or division of decimals including metric or money values.
48. Solve word problems requiring the calculation of final cost when the discount or sales tax rate and the items' costs are given.
49. Find amount of discount, commission, or down payment when the rate and total cost are given.
50. Find averages to hundredths.
51. Solve word problems involving a total and the number of items totaled, and requiring the calcualtion of the average to hundredths.
52. Solve word problems involving time.
53. Solve word problems using information obtained from tables and charts.
54. Determine change from a given amount of money on a purchase of two or more items.
55. Solve word problems describing a realistic situation with no more than five items, addends up to $99.99, and amount tendered up to $200.00.
56. Solve problems comparing unit costs and rates to determine the best purchase or determine the amount of saving when the same item is offered at different prices.
57. Solve word problems that compare that compare the cost of cash and credit purchases.

NOTES FOR THE TEACHER

These outcomes are discussed thoroughly in the mathematics section of the Alabama High School Graduation Exam Basic Skills Instructional Materials (BSIM). Any further development of these outcomes would be a natural extension of the materials contained in that document.

OUTCOME 47 - BSIM Competency I-4
OUTCOMES 48 and 49 - BSIM Competency I-7
OUTCOMES 50 and 51 - BSIM Competency I-9
OUTCOME 52 - BSIM Competency II-1
OUTCOME 53 - BSIM Competency III-2
OUTCOMES 54 and 55 - BSIM Competency VI-1
OUTCOME 56 - BSIM Competency VI-2
OUTCOME 57 - BSIM Competency VI-3

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Multiply by one-, two-, and three-digit factors				
8			Divide by one-, two, and three-digit divisors				
10			Add, subtract, multiply, and divide with whole numbers, fractions, and decimals				
		I-3	Add, subtract, multiply, and divide whole numbers				
	*		Addition	1. Solve addition problems with no more than five addends.			
				2. Solve addition problems with addends up to five digits.			
				3. Solve addition problems with or without regrouping.			
				4. Compute sums presented in horizontal or vertical format.			
	*		Subtraction	5. Subtract numbers with subtrahends and minuends of up to five digits with or without regrouping.			
				6. Solve subtraction problems presented in horizontal or vertical format.			

MATHEMATICS: GENERAL AREA: WHOLE NUMBERS - COMPUTATION (Cont'd)

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Multiplication	7. Multiply one-, two-, three-digit factors.			
				8. Solve multiplication problems presented in horizontal or vertical format.			
	*		Division	9. Solve division problems with dividends up to five digits with or without remainders.			
				10. Solve division problems presented symbolically by: $a \div b$ or b/a .			
				11. Interpret and solve division problems stated in fractional format, i. e., $\frac{a}{b}$.			

MATHEMATICS: GENERAL

AREA: WHOLE NUMBERS - ESTIMATION AND APPROXIMATION

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Estimate products and quotients of whole numbers and decimal numbers				
		IV-1	Make approximations by rounding numbers				
	*		Estimation and approximation	12. Interpret place value in numerals of up to seven digits and round numbers.			
				13. Estimate sums, difference, products, and quotients of whole numbers.			
				14. Estimate a sum with up to five addends.			
				15. Round whole numbers to tens, hundreds, and thousands.			
				16. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.			

MATHEMATICS: GENERAL AREA: WHOLE NUMBERS - POWERS AND SQUARE ROOTS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Demonstrate an understanding of the fundamental operations and their properties				
	*		Raising to a power	17. Read powers of numbers and find the products they indicate.			
				18. Identify the expanded form of a number expressed in exponential form.			
				19. Subtract a number less than ten from a number expressed in exponential form.			
	*		Finding of the root of a number	20. Identify the square roots of perfect squares.			
				21. Find square roots using a table of square roots.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8 10			Solve problems, express problems as number sentences, and identify missing or irrelevant data				
	*		Applications involving whole numbers	22. Solve one-step and two-step problems.			
				23. Solve word problems using information obtained from picto-, bar, line, and circle graphs.			
				24. Solve rate problems.			
				25. Identify the number sentence appropriate to solving a two-step problem.			
				26. Identify the information needed to solve a problem.			
				27. Identify the information not needed to solve a problem.			
		I-9	Find averages of whole numbers and decimal numbers	28. Solve a problem requiring the calculation of an average.			
		VI-6	Solve rate, time, distance problems	29. Solve word problems to determine miles per gallon when given distance in miles and gallons of fuel used.			
				30. Solve word problems to determine time when given distance traveled and average rate of speed.			
				31. Solve word problems to determine rate when given distance traveled and time.			

MATHEMATICS: GENERAL

AREA: WHOLE NUMBERS - APPLICATIONS (Cont'd)

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
		VI-7	Compute salaries and wages	32. Solve word problems involving computation of net pay when salary and deductions are given in only dollar amounts.			
				33. Solve word problems involving computation of net pay when amounts of deductions, the rate of pay, and the number of hours worked are given.			
		III-1	Read and interpret graphs	34. Solve word problems requiring the interpretation of picto-, bar, line, or circle graphs and the use of information contained in the graphs.			

331

332

MATHEMATICS: GENERAL AREA: DECIMALS - COMPARE AND ORDER

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
		I-2	Compare two decimals	1. Identify place value in decimal numbers and mixed decimals up to six digits. 2. Compare decimal numbers between zero and one to thousandths. 3. Select from the options a decimal less than or greater than a given decimal.			

MATHEMATICS: GENERAL AREA: DECIMALS - COMPUTATION

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Add, subtract, multiply, and divide with decimals				
10			Add, subtract, multiply, and divide with whole numbers, fractions, and decimals				
		I-4	Add, subtract, multiply, and divide decimals				
	*		Addition	4. Add two or more decimal numbers.			
				5. Add decimal numbers and whole numbers.			
				6. Add decimal numbers and mixed decimals.			
				7. Add whole numbers and mixed decimals.			
				8. Add two or more mixed decimals.			
				9. Add sums involving up to three unlike decimal addends.			
				10. Add decimal numbers to two places with regrouping and with no more than five digits to the left of the decimal.			
				11. Add sums involving money values.			
				12. Compute sums presented in horizontal or vertical format.			

S T A N	C OF S	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	*		Subtraction	13. Subtract one decimal number from another.			
				14. Subtract a whole number from a decimal number and vice versa.			
				15. Subtract a decimal number from a mixed decimal.			
				16. Subtract one mixed decimal from other.			
				17. Subtract decimal numbers to two places with regrouping.			
				18. Subtract money values.			
				19. Subtract decimal numbers with no more than five digits to the left of the decimal.			
				20. Solve subtraction problems presented in horizontal or vertical format.			
	*		Multiplication	21. Multiply: whole numbers and decimal numbers; whole numbers and mixed decimals; a decimal number by another decimal number; decimal numbers and mixed decimals; and a mixed decimal by another mixed decimal.			
				22. Multiply a decimal by a power of ten by relocating the decimal point.			
				23. Multiply hundredths times tenths.			
				24. Multiply hundredths times hundredths.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				25. Multiply money values.			
				26. Multiply decimals with no more than three digits to the left of the decimal.			
				27. Multiply decimals numbers presented in horizontal or vertical format.			
	*		Division	28. Divide a decimal number or a mixed decimal by a whole number, correctly placing the decimal point in the quotient.			
				29. Express the quotient of two whole numbers as a decimal number.			
				30. Divide by powers of ten by relocating the decimal point.			
				31. Rewrite and solve a division problem with a decimal number or a mixed decimal divisor as an equivalent problem with a whole number divisor.			
				32. Divide hundredths by tenths.			
				33. Divide tenths by whole numbers of not more than two digits.			
				34. Divide decimal numbers with no more than two digits to the left of the decimal in the division.			
				35. Divide decimal numbers with dividends not to exceed five digits.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
		I-6	Change percents to decimals and decimals to percents	36. Select a whole percent up to 200% equivalent to a given decimal number up to two places.			
				37. Select a decimal number up to two places equivalent to a given percent up to 200%.			
				38. Rename a decimal fraction or a mixed decimal, up to three digits, as a percent or vice versa.			
		I-8	Convert fractions to decimals and decimals to fractions	39. Select a fraction or a mixed number equivalent to a given decimal.			
				40. Select a decimal equivalent to a given fraction or mixed number.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Estimate products and quotients of whole numbers and decimal numbers				
10			Estimate products and quotients of decimal numbers and square roots				
		IV-1	Make approximations by rounding numbers				
	*		Estimation and approximation	41. Identify place value in decimal numbers and mixed decimals up to six digits.			
				42. Round decimal numbers and mixed decimals to a specified place value.			
				43. Estimate to the nearest whole number sums, differences, products, and quotients involving decimal numbers or mixed decimals.			
				44. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.			
				45. Estimate to the nearest ten cents or dollar.			
				46. Round to the nearest dollar.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Applications involving decimals				
		I-4	Add, subtract, multiply and divide decimals	47. Solve word problems involving realistic applications requiring addition, subtraction, multiplication, or division of decimals including metric or money values.			
		I-7	Determine discounts, down payments, commissions, taxes, ratios, and percents	48. Solve words problems requiring the calculation of the final cost when the discount or sales tax rate and the items´ costs are given.			
				49. Find amount of discount, commission, or down payment when the rate and total cost are given.			
		I-9	Find averages of whole numbers and decimal numbers	50. Find averages to hundredths.			
				51. Solve word problems involving a total and the number of items totaled and requiring the calculation of the average to hundredths.			
		II-1	Solve problems involving time	52. Solve word problems involving time.			
		III-2	Read and interpret tables and charts	53. Solve word problems using information obtained from tables and charts.			
		VI-1	Determine change	54. Determine change from a given amount of money on a purchase of two or more items.			
				55. Solve word problems describing a realistic situation with no more than five items, addends up to $99.99, and amount tendered up to $200.00.			

MATHEMATICS: GENERAL AREA: DECIMALS - APPLICATIONS (Cont´d)

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
		VI-2	Use comparative shopping skills	56. Solve problems comparing unit costs and rates to determine the best purchase, or determine the amount of savings when the same item is offered at different prices.			
		VI-3	Compare the cost of cash and credit purchases	57. Solve word problems that compare the cost of cash and credit purchases.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Name, order, and interpret fractions				
10			Interpret fractions, decimals, percents, and integers				
		I-2	Compare decimals; compare fractions with unlike denominators	1. Identify the fraction that is equivalent to a given common fraction.			
				2. Identify the common fraction that is expressed in simplest form.			
				3. Rename a common fraction to simplest form.			
				4. Rename a mixed number as an improper fraction.			
				5. Find the least common denominator for three common fractions.			
				6. Identify the greatest of four common fractions with unlike denominators.			
				7. Order three common fractions with unlike denominators from least to greatest.			
				8. Identify the proper fractions that are less than or greater than a given proper fraction.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Add, subtract, multiply, and divide with fractions.				
10			Add, subtract, multiply, and divide with whole numbers, fractions, and decimals.				
		I-5	Add, subtract, and multiply fractions				
	*		Addition	9. Add two common fractions with like denominators with renaming.			
				10. Add three common fractions with unlike denominators.			
				11. Add two mixed numbers with unlike denominators.			
				12. Add proper fractions and mixed numbers with unlike denominators.			
	*		Subtraction	13. Subtract one mixed number from another with a like denominator with renaming.			
				14. Subtract a common fraction from a mixed number, each with a different denominator.			
				15. Subtract proper fractions and mixed numbers with unlike denominators.			
	*		Multiplication	16. Multiply a common fraction by a whole number.			
				17. Multiply two common fractions with renaming.			

MATHEMATICS: GENERAL

AREA: FRACTIONAL NUMBERS - COMPUTATION (Cont'd)

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				18. Multiply two mixed numbers.			
				19. Multiply two factors of proper fractions and/or mixed numbers.			
	*		Division	20. Divide a whole number by a common fraction.			
				21. Divide a common fraction by a whole number.			
				22. Divide a mixed number by a whole number.			

MATHEMATICS: GENERAL AREA: FRACTIONAL NUMBERS - CONVERSION TO DECIMALS AND PERCENTS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Name, order, and interpret fractions, decimals, percents, and integers				
10			Interpret fractions, decimals, percents, and integers				
		I-8	Convert fractions to decimals and decimals to fractions	23. Rename a common fraction expressed in thousandths as a decimal fraction.			
				24. Rename a mixed decimal as a mixed number in simplest form.			
				25. Rename a decimal fraction as a common fraction in simplest form.			
				26. Rename a percent as a common fraction in simplest form.			
				27. Rename a fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 or a mixed number less than ten to a given decimal.			
				28. Rename a decimal equivalent to a given fraction with denominators of 2, 4, 5, 8, 10, 100, or 1000 to a mixed number less than ten.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Estimation and Approximation	29. Make approximations by rounding to whole numbers, proper fractions stated in simplest form and with single digit denominators.			
				30. Make approximations by rounding to whole numbers, mixed numbers stated in simpest form and with single digit denominators.			
				31. Estimate sums, differences, products, and quotients involving fractional numbers.			
				32. Solve word problems using the principle of rounding to find approximate sums, differences, products, and quotients.			

MATHEMATICS: GENERAL AREA: FRACTIONAL NUMBERS - BASIC OPERATIONS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Basic operations involving mixed numerals	33. Change a mixed number to an improper fraction.			
				34. Change an improper fraction to a mixed number.			
				35. Add two or more unlike mixed numbers with or without renaming.			
				36. Add like or unlike mixed numbers and fractions or whole numbers with or without renaming.			
				37. Subtract a mixed number, a fraction, or a whole number from a like or unlike mixed number with or without renaming.			
				38. Subtract a mixed number or a proper fraction from a whole number with or without renaming.			
				39. Multiply mixed numbers with or without renaming.			
				40. Divide mixed numbers with or without renaming.			
	*		BCT restrictions applying to basic opertions involving mixed numerals are stated in student outcomes for addition, subtraction, and multiplication of fractional numbers.				

MATHEMATICS: GENERAL AREA: FRACTIONAL NUMBERS - APPLICATIONS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Applications involving fractional numbers	41. Recognize the proper operation needed to solve practical problems involving addition, subtraction, and multiplication of fractions.			
				42. Solve practical problems involving addition, subtraction, and multiplication of proper fractions and mixed numbers.			
				43. Solve a one-step division problem in which the answer is expressed as a fraction in simplest form.			
				44. Solve a rate problem involving a fraction or a mixed number.			

S T A N	C OF S	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8			Interpret integers				
10			Interpret fractions, decimals, percents, and integers				
	*		Meaning and representation of integers on a number line	1. Demonstrate an understanding of the definition by listing examples of the set of integers. 2. Given a number line, identify specific integers. 3. Given an integer, state its opposite. 4. Define absolute value as the distance a number is from zero on the number line. 5. State the integer corresponding to a given absolute. 6. Simplify expressions that involve absolute value. 7. Identify the integer that is between two given integers. 8. Write inequalities using positive and negative integers.			

MATHEMATICS: GENERAL AREA: INTEGERS - COMPUTATION

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Name, order, and interpret fractions, decimals, percents, and integers.				
10			Demonstrate an understanding of whole numbers and rational numbers and of operations and their properties.				
	*		Addition	9. Add positive integers.			
				10. Add negative integers.			
				11. Add positive and negative integers.			
	*		Subtraction	12. Subtract positive integers.			
				13. Subtract negative integers.			
				14. Subtract positive and negative integers.			
	*		Multiplication	15. Multiply positive integers.			
				16. Multiply positive and negative integers.			
				17. Multiply negative integers.			
	*		Division	18. Divide positive integers.			
				19. Divide a positive integer by a negative integer and vice versa.			
				20. Divide negative integers.			

MATHEMATICS: GENERAL AREA: INTEGERS - APPLICATIONS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Applications involving integers	21. Translate word problems into expressions using integers. 22. Solve word problems involving integers.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Demonstrate an understanding of the principles of geometry and measurement				
10			Apply the principles of geometry and measurement				
		V-1 V-3	Recognize simple plane geometric figures: parallel and perpendicular lines				
		V-2	Recognize common solid geometric figures				
	*		Geometry of two dimensions	1. Identify and name points, lines, planes, segments, rays, intersecting lines, and parallel lines.			
				2. Measure and classify angles as acute, right, or obtuse.			
				3. Identify congruent polygons and corresponding parts of the congruent polygons.			
				4. Classify triangles in terms of their sides and angles.			
				5. Classify polygons in terms of their sides and angles.			
				6. Recognize that the sum of the measures of the angles of the triangle is 180°.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				7. Identify center, chord, diameter, radius, arc, central angle, and inscribed angle of a circle.			
				8. Select the figure or state the names of the following plane geometric figures or lines: angle, right angle, triangle, square, rectangle, circle, lines, perpendicular lines, or parallel lines.			
				9. Identify prisms, pyramids, cylinders, cones, and spheres.			
	*		Geometry of three dimensions	10. Select the figure or state the name of the following solid geometric figures: sphere, cube, rectangular solid, cylinder, and cone.			

MATHEMATICS: GENERAL

AREA: INFORMAL GEOMETRY - APPLICATIONS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Applications of Geometry	11. Evaluate formulas.			
				12. Find the perimeter or circumference of a shape.			
		II-4	Find perimeters	13. Find the perimeter of polygons up to six sides and units expressed in inches, feet, yards, centimeters, or meters.			✓
				14. Find the perimeter of rectangles, squares, and triangles expressed in word problems.			
				15. Find the perimeter of polygons when all dimensions of the figure are given.			

MATHEMATICS: GENERAL AREA: LINEAR EQUATIONS AND INEQUALITIES - SOLVING EQUATIONS

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Demonstrate the ability to solve proportions and linear equations				
10			Solve proportions and linear equations				
	*		The solving of equations by addition The solving of equations by subtraction The solving of equations by multiplication The solving of equations by division	1. Use the standard order of operations to evaluate an expression involving more than one operation. 2. Evaluate an expression for a given value of the variable. 3. Evaluate a linear equation. 4. Solve an addition or subtraction equation by subtracting the same number from both sides or by adding the same number to both sides. 5. Solve a multiplication or division equation by dividing or multiplying both sides by the same number.			
	*		The solving of equations involving more than one operation	6. Solve two-step equations.			

MATHEMATICS: GENERAL

AREA: LINEAR EQUATIONS AND INEQUALITIES - PROBLEM SOLVING

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Problem solving involving equations	7. Write an expression involving a variable for a given mathematical phrase or situation. 8. Solve problems by writing equations involving addition, subtraction, multiplication, or division and solving the equation.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		The solving of linear inequalities	9. Graph inequalities on the number line.			
				10. Solve an addition or subtraction inequality involving rational numbers.			
				11. Solve a multiplication or a division inequality involving rational numbers.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Apply an understanding of geometric properties to measurement and apply an understanding of units of measure				
10			Apply the principles of geometry and measurement				
	*		Linear units	1. Recognize equivalent customary units.			
				2. Recognize equivalent metric units.			
		II-2	Measure distance by use of ruler	3. Measure line segments or objects in a figure in customary and metric units.			
				4. Measure line segments or objects in figure in customary and metric units requiring the addition of two lengths.			
	*		Area	5. Find the area of a rectangle, square, parallelogram, triangle, trapezoid, and a circle.			
				6. Find the surface area of a prism, a pyramid, a cylinder, a cone, and a sphere.			
		II-5	Find rectangular areas	7. Solve word problems relating to rectangular area involving customary or metric units of inches, feet, yards, centimeters, or meters.			
				8. Solve problems requiring the calculation of rectangular area when figures are used and lengths of all sides are labeled.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Volume	9. Find the volume of a prism, a pyramid, a cylinder, a cone, and a sphere when the dimensions are given in customary or metric units.			
	*		Mass	10. Recognize equivalent customary units.			
				11. Recognize equivalent metric units.			
				12. Choose the appropriate customary measure of mass and find equal measures for given measures of mass.			
				13. Choose the appropriate metric measure of mass and find equal measures for given measures of mass.			
				14. Add and subtract with metric measures.			
				15. Add and subtract with customary measures.			
	*		Capacity (liquid)	16. Recognize equivalent customary units.			
				17. Recognize equivalent metric units.			
				18. Choose the appropriate customary measure of capacity and find equal measures for given measures of capacity.			
				19. Choose the appropriate metric measures of capacity and find equal measures for given measures of mass.			
				20. Add and subtract with metric measures.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				21. Add and subtract with customary measures.			
	II-3		Measure capacities	22. Solve word problems involving capacities with units of ounces, pints, quarts, gallons, milliliters, or liters.			
				23. Solve word problems involving capacity but without conversions between metric and customary units and without using formulas to determine volume.			
	*		Temperature	24. Identify the measuring scale for temperature in metric measure as degrees Celsius.			
				25. Identify the measuring scale for temperature in customary measure as degrees Fahrenheit.			
				26. Add and subtract with metric measures.			
				27. Add and subtract with customary measures.			
				28. Solve word problems involving computation of metric measures or customary measures.			
	*		Practical applications	29. Choose the appropriate metric measure for linear, mass, or capacity measure, for a given situation.			
				30. Choose the appropriate customary measure for linear, mass, or capacity measure, for a given situation.			
				31. Solve problems by giving sensible answers.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				32. Solve word problems using the appropriate customary or metric measures.			
				33. Add and subtract measures of time and compute elapsed time.			
	II-1		Solve problems involving time	34. Solve problems using addition, subtraction, multiplication, and division for units of time to include years, months, weeks, days, hours, and minutes.			
				35. Solve problems using fractional units of time involving denominators of 2, 3, 4, 6, 10, and 12.			
	I-10		Change units of measure	36. Change units of linear, capacity, and weight measures by converting within the customary system.			
				37. Change units of linear, capacity, and weight measures by converting within the metric system.			
				38. Convert linear customary units of measure of inches, feet, and yards or metric units of measures of millimeters, centimeters, meters, and kilometers.			
				39. Convert measures of customary capacity units of quarts and gallons or metric capacity units of milliliters and liters.			

MATHEMATICS: GENERAL AREA: MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS (Cont'd)

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
				40. Convert measures of customary weight units of ounces and pounds or metric weight units of grams and kilograms.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Solve problems, express problems as number sentences, and identify missing or irrelevant data				
10			Demonstrate the ability to solve problems, to interpret graphed functions, and to understand and apply the principles of geometry and measurement				
		III-1	Read and interpret graphs				
		I-9	Find averages of whole numbers and decimal numbers				
	*		The gathering of data including sampling	1. Make a predicition for an outcome. 2. Determine probability using experiments.			
	*		Analysis of data	3. Use a graph or a table to organize data and determine the range for a set of data. 4. Identify the mean, the median, and the mode for a set of data. 5. Given a frequency table, determine the number of cases on which a score distribution is based. 6. Given a frequency table, name the number of scores at or below a given score.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		The reporting of data including hisograms and graphs	7. Solve word problems requiring the interpretation of a picto-, bar, line, or circle graph and the use of information contained in the graph.			
				8. Read and construct tables or graphs used to organize data.			
	*		Probability of an event	9. Find the probability of single and multiple outcomes.			
				10. Determine the probability of independent and dependent events.			
				11. Make a prediction for an outcome.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
8			Demonstrate the ability to solve proportions and linear equations				
10			Solve proportions and linear equations				
		I-6	Change percents to decimals and decimals to percents				
		I-7	Determine discounts, down payments, commissions, taxes, ratios, and percents				
		III-3	Interpret scale drawings				
	*		The use of ratios and proportions to solve problems	1. Solve word problems requiring the calculation of the ratio of two numbers.			
				2. Solve a proportion in which the unknown is a numerator in one ratio and a denominator in the other ratio.			
				3. Solve a proportion in which a denominator is unknown.			
	*		Scale drawings	4. Solve word problems relating to scale drawings using whole numbers.			
				5. Use scale drawings and proportions to determine actual lengths.			

S T A N	C OF S	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	*		Percent as a ratio	6. Write decimals as percents and percents as decimals. 7. Demonstrate an understanding that a percent is actually the ratio of a number to 100.			

STAN	C OF S	BCT	ORIGINAL SOURCES	STUDENT OUTCOMES	BAS	STAN	ADV
	*		Money problems				
		I-7	Determine discounts, down payments, commisions, taxes, ratios, and percents	1. Solve word problems requiring the calculation of: the final cost when the discount, or sales tax rate, and item costs are given; the amount of discount, tax, commision, or down payment when the rate and total cost are given; or the ratio of two whole numbers.			
		VI-1	Determine change	2. Solve word problems requiring: calculation of of the total purchase price and determination of correct change; determination of change when the purchase price and the amount tendered are given; recognition of equivalent amounts using different combinations of coins and currency.			
		VI-2	Use comparative shopping skills	3. Solve word problems requiring: the selection of the best of four purchases of the same item; determination of savings when the same item is offered at different prices.			
		VI-3	Compare the cost of cash and credit purchases	4. Solve word problems requiring: computation of the interest charged on a credit purchase; computation of the price difference between a credit purchase and a cash purchase; comparing cost of a cash and credit purchase from the information given in an advertisement.			
		VI-4	Allocate time and money	5. Solve word problems requiring: solution of realistic problems involving time and money; use of information given in chart form.			

S T A N	C OF S	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
		VI-5	Complete checks and money orders	6. Given a blank check or money order, determine the correct placement of a given item of information.			
				7. Given a completed check or money order, determine incorrect placement or error in placement of information.			
		VI-7	Compute salaries and wages	8. Solve word problems requiring: computation of net pay when salary and deductions are given; computation of net pay when amount of deductions, the rate of pay, and the number of hours worked are given.			
	*		Distance problems				
		VI-6	Solve rate, time, distance problems	9. Solve word problems for: miles per gallon when given distance in miles and gallons of fuel used; time when given distances traveled and average rate of speed; rate when given distance traveled and time.			